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MULTIPLE BRINE PRODUCTION AND STORAGE FACILITY LOCATION PROBLEM FOR SNOW REMOVAL

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**MULTIPLE BRINE PRODUCTION AND STORAGE
FACILITY LOCATION PROBLEM FOR SNOW REMOVAL**

BY

MORITZ BEHREND

**A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE**

IN

INDUSTRIAL AND SYSTEMS ENGINEERING

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OF

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ABSTRACT

In a departure from past practice, snow plow operations are now executed in a two phase approach: a preparation phase where brine is sprayed on roads, and a snow removal phase. In order to augment existing snow removal operations by the first phase, investments in new facilities such as silos for brine storage, and equipment such as brine spreaders are required. To maximize the impact of these investments, decisions such as the optimal location of silos and the selection of appropriate equipment must be carefully considered. This thesis addresses the facility location problem only.

The benefit of investing in facilities and assigning them to locations in a network is assessed based on two premises: operations shall be expedited and brine shortage shall be avoided. Operations are expedited by providing more replenishment points for trucks so that the time for replenishment dwindles. The model is formulated as a deterministic facility location model.

The second premise takes into account that silos are cheaper than brine machines but that they are also less effective in averting brine shortage. Since they are cheaper, more replenishment points can be set up and solving the model only based on the first premise would always suggest to invest in silos. But when subsequently hitting snow storms require plenty of operations with only a few days in between, silos eventually deplete and brine shortage is imminent. The deterministic model is therefore extended to a scenario-based model incorporating variability due to weather. All possible facility combinations, so-called facility type mixes, undergo a stress test and it is

assumed that when no brine shortage occurred during this stress test, the facility type mix is also appropriate for less intense weather scenarios which were not considered.

In this work, a new concept is assessed which comprises a brine machine on a boat. Such a boat helps to avert brine shortage and additionally provides a mobile replenishment point for trucks.

The model is applied to a case study for the Rhode Island Department of Transportation.

Keywords: Snow Removal, Brine Operations, Scenario-based Optimization, Facility Location Problem

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LIST OF ABBREVIATIONS

C	Celsius
cf.	confer
FLP	Facility Location Problem
gals.	Gallons
GB	Gigabyte
GHz	Gigahertz
i.e.	id est
NOAA	National Oceanic and Atmospheric Administration
RAM	Random Access Memory
RIDOT	Rhode Island Department of Transportation
RO	Robust Optimization

LIST OF SYMBOLS

Sets

$a \in \mathcal{A}$	Water alternatives.
$b \in \mathcal{B}$	Prospective brine machine locations.
$g \in \mathcal{G}$	Grid points.
$i \in \mathcal{I}$	Primary flows.
$j \in \mathcal{J}$	Secondary flows.
$k \in \mathcal{K}$	Unique scenarios.
$m \in \mathcal{M}_w$	Mobile silos associated with w .
$n \in \mathcal{N}$	Weather stations.
$p \in \mathcal{P}$	Periods.
$s \in \mathcal{S}$	Prospective silo locations.
$t \in \mathcal{T}$	Silo size add-ons.
$w \in \mathcal{W}$	Water stations.

Parameters

$aACOST$	Activation costs of a .
$ACOST_t$	Activation costs of t .
$AddSCAPA_t$	Additional storage capacity associated with t .
$aPCAPA$	Production capacity of a .
$aSCAPA$	Storage capacity of a .
$bACOST$	Activation costs of b .
$BigM$	Large number.

$bPCAPA$	Production capacity of b .
$budget$	Initial budget for investments.
$clusters$	Defined quantity of clusters.
DEM	Brine demand of i or j in p .
$DIST$	Distance between a facility and another facility, a facility and a flow, or a grid point and a weather station.
$events_{g_1g_2}$	Unequal weather events between g_1 and g_2 .
$MaxDIST$	Maximum distance between g and n .
$MinServiceLevel$	Minimum accepted service level of secondary flows.
p_k	Frequency of occurrence of k .
$sACOST$	Activation costs of s .
$sBaseSCAPA$	Base storage capacity of s .
$valid_{np}$	Valid weather information for n in p .
z_k^*	Optimal value for k .

Variables

α	Activation of a , b , s , or st , activation of w in p , or activation of g as center.
δ	Storage level of a or s in p .
κ	Assignment of g_1 to g_2 or assignment of g to n in p .
ρ	Brine production of a or b in p .
ς_{gp}	Slack for g in p .
χ	Brine delivered from a facility to another facility or a flow in p .
ζ_k	Objective value for k .

Chapter One

INTRODUCTION

1.1 Snow Removal Operations

In winter, snow removal operations have a substantial impact on transportation. They restore safe road conditions and are therefore indispensable. However, they also occasion major costs for labor, travel, and supplies such as abrasives and equipment.[cf. Perriera, Langevin, et al. 2005, p. 212 and cf. Perriera, Langevina, et al. 2004, p. 210] Jang et al. 2010 show that expenditures for labor, travel, and equipment can be reduced. Using the minimum required fleet size for maintaining a specified service level and setting up depots at strategic locations in the network of Boone County, Missouri, a 26% reduction of operational costs is said to be possible. Abrasives are not the subject of their work, although they are a big cost driver, as well. For instance, chemical abrasives are expensive and their application leads to consequential costs such as repair works for damages to roads and bridges. Bearing in mind that an excessive use of chemicals also leads to environmental problems such as contaminated soil or water, it is required not only to improve but to rethink existing procedures to remove snow from roads.

Snow removal operations can be categorized as chemical, mechanical, and thermal operations.[cf. Perriera, Langevina, et al. 2004, p. 12] By using chemicals, snow turns into slush and eventually may melt completely, depending on the amount and effectiveness of the applied chemicals and the

amount of snow. Mechanical operations such as plowing remove snow from roads and pavements by using force, whereas thermal operations melt snow by using heat. An example for thermal operations is to place sewer pipes, which emit heat, underneath pavements so that their surface temperature stays above 0°C. Mechanical and thermal snow removal operations are not covered in this work. The reader is referred to Minsk 1998.

The application of chemicals is either used to facilitate snow removal or to remove snow by itself. The former case is called anti-icing and requires the preparation of roads prior to snowfall so that when snow turns into slush, it cannot pack to the asphalt and is easier to remove. The latter case, deicing, is required when snow or ice has already packed to the asphalt. Since packed snow or ice on asphalt can have severe implications for traffic, road maintenance should primarily focus on anti-icing measures.

A commonly used chemical is salt, due to its wide availability, its low price, and its good snow melting characteristics at temperatures close to 0°C.[Perriera, Langevina, et al. 2004, p. 212] When salt is applied as an anti-icing measure, a negative aspect is that it easily gets blown off the street and the measure fails to serve its purpose. Salt is wasted and the environment is strained unnecessarily. As a solution, salt can be mixed with water to make brine prior to application. Once brine is sprayed on a dry street, the water evaporates and the salt remains. So, regardless of whether salt or brine is used as an anti-icing measure, in the end salt remains on the road and the anti-icing effect stays the same. The difference is, however, that salt sticks to the asphalt when applied with water. It is also distributed more evenly, since salt completely dissolves in water, and less salt is required. Aside from these advantages, there are also disadvantages. It is important to note that brine cannot be applied at the same temperatures as salt. Its freezing point, although dependent on the salt concentration, can at minimum be -21.1°C

with a salt concentration of 23.3%.[Varitech Industries 2013] Consequently, brine is only an alternative for temperate regions since the least desired effect is that brine turns into ice. Another disadvantage is that in order to augment existing snow removal operations, investments in facilities for brine production and storage are required.

1.2 Methodology and Procedures

In section 1.1 it is described that an excessive use of chemicals for snow removal can be reduced by using brine instead of salt. But unlike salt, which does not require special storage facilities except for a shelter, brine must be stored in non-corrosive storage tanks. Additionally, production facilities are required to produce brine. The goal of this thesis is to develop recommendations so that the impact of these investments is maximized in regard to the following two premises:

1. Locations for new facilities should expedite snow removal operations.
2. Facility type selection and quantity of facilities should minimize the likelihood of brine shortage.

The first premise deals with the aspect that tanks on brine trucks have a finite capacity. By investing in either production or storage facilities, the availability of brine increases in a network and the time a truck travels to replenish its brine tank decreases. Unproductive travelling, in which no brine is spread, is called dead mileage and, provided that the street network is several times larger than the maximum capacity of a truck, can be a major waste of time. Therefore, the first premise is addressed by locating facilities strategically, meaning that facilities are assigned to areas in which high demand occurs but replenishment opportunities are rare.

The second premise addresses the facility type selection. Silos have a certain capacity and can be sited almost everywhere since they do not require any special infrastructure. Once a silo is depleted, money, time, and brine are required to replenish it. In contrast, production facilities, which are in fact relatively small brine machines, have a certain production capacity and cannot be located everywhere because they require energy, water, and salt to produce brine. The set of prospective brine machine locations is therefore smaller than the set of prospective silo locations. Since silos are significantly cheaper than brine machines, investments in silos are more reasonable in regard to the first premise because more replenishment points can be set up. However, silos and brine machines are differently effective in averting brine shortage and a facility location problem for snow removal should take this aspect into account. For instance, having frequent snow storms with only a few days of fair weather might make it difficult to replenish all silos on time. In such cases, brine shortage could be imminent and a brine machine might be a better choice.

Both premises require information on the distribution of brine demand, which is contingent on weather. Therefore, weather records are incorporated in the decision-making process. Since brine production and storage facilities are only utilized for brine operations, weather is the only source of variability.

The structure of the work is as follows: chapter 2 gives background knowledge for both network location problems and robust optimization, presents what is already available, and explains where this work continues. Based on this chapter, chapters 3 and 4 develop approaches to address the problem stated in chapter 1. Section 3.1 develops a static facility location model in which facilities are to be located such that the difference between set-up costs and the benefit of setting up a facility is optimized; section 3.2

presents a new concept in which a mobile brine machine is considered. Since this concept is a novelty, its implementation and feasibility are explained, as well. Chapter 4 explains how to incorporate weather data to analyse the robustness of a brine production and distribution network against brine shortage. In the following chapter 5, the devised model is applied to a case study for the Rhode Island Department of Transportation and recommendations are developed. This work concludes with a summary and an outlook for future work in chapter 6. An overview of this work is depicted in figure 1.1.

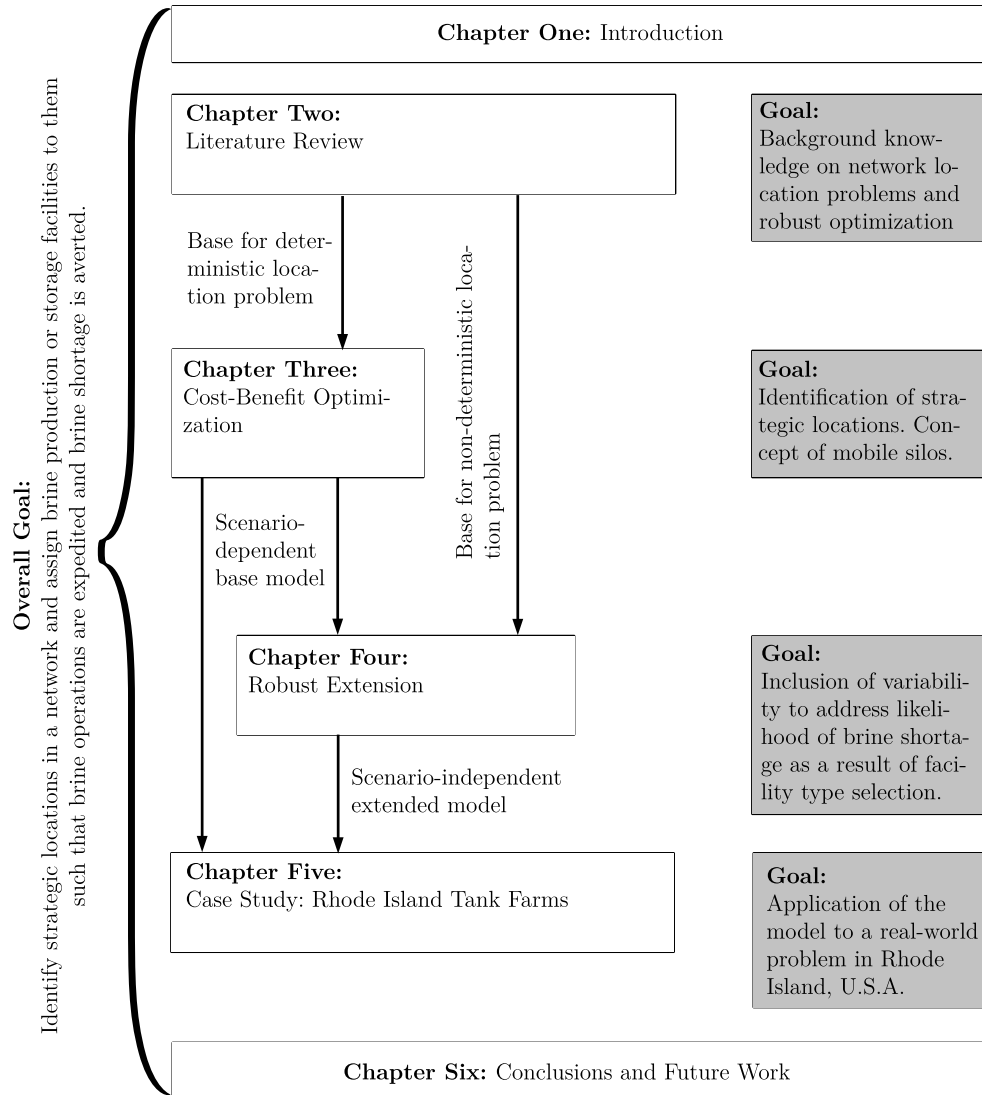


Figure 1.1: Overview of this work.

Chapter Two

LITERATURE REVIEW

Recommendations for augmenting existing snow removal operations with brine as an anti-icing measure are devised based on the two premises stated in chapter 1. First, facilities should be located such that spreading operations are expedited and second, the likelihood of brine shortage should be minimized. The focus of this chapter is to present background knowledge on both types of problems, such as notation and basic concepts, but also to present already available models and approaches. This chapter corresponds to the base of this work upon which the models in chapters 3 and 4 are built.

2.1 Network Location Problems

This section introduces networks and facility location problems (FLPs). Since locating facilities implies working with networks, an introduction to graph theory is given first. In the second subsection, the FLP in this work is defined and its scope is determined. Afterwards, existing models are discussed.

2.1.1 Introduction to Graph Theory

A network is described by a graph of interconnected nodes, whereas the connection between two nodes corresponds to their relation. Based on the type of connections, a graph can either be directed or undirected. In the former case, nodes are connected via arcs and they can only be travelled in

one direction; in the latter case, connections between nodes are established via bidirectional edges. Since the basis of snow removal related problems is a street network, only the directed graph is used throughout this work.

More formally, a directed graph is described as $G = (N, A)$, with N being the set of n nodes and A being the set of m arcs.[cf. Ahuja et al. 1994, p. 2] A relation between two nodes i, j is established by $(i, j) \in A$ and is expressed by c_{ij} , whereas c_{ij} represents costs for travelling along an arc. It is assumed that costs increase linearly with the amount of flow. Upper and lower bounds for arcs can be imposed with u_{ij} and l_{ij} , respectively. To take advantage of the graph structure, each node must be associated with a demand or a supply of a certain good, expressed by $b_i \forall i \in N$. If $b_i > 0$, node i is a supply node; if $b_i < 0$, node i is a demand node; and if $b_i = 0$, node i is a transshipment node. In the following, only those aspects of graph theory are described which are relevant for this work. The interested reader is referred to Bondy 2008, Ahuja et al. 1994, or Guisewite and Pardalos 1990.

Setting up a graph implies that modifications might be required in order to work with it. Therefore, some valid transformations are presented here before further notations and the concept of minimum cost flows are introduced. Edges can be transformed into arcs by replacing them with two opposite directing arcs. Splitting a node i in $i', i'' \in N$ is valid, as well, as long as all connections between node i and contiguous nodes $j : (i, j), (j, i) \in A$ remain. Splitting nodes is useful for distinguishing between incoming and outgoing flows of a node. To ensure that node i' and i'' still represent one node, new connections $(i', i''), (i'', i') \in A$ with costs $c_{i', i''} = c_{i'', i'} = 0$ and infinite upper bounds are established.[cf. Ahuja et al. 1994, p. 2]

A path in $G = (N, A)$ is a sequence of nodes and arcs $i_1, (i_1, i_2), i_2, (i_2, i_3), \dots, (i_{r-1}, i_r), i_r$ satisfying the property that $(i_k, i_{k+1}) \in A \forall k = 1, \dots, r-1$. All nodes must be distinct. A graph $G' = (N', A')$ is a subgraph

of $G = (N, A)$ if $N' \subseteq N$ and $A' \subseteq A$. A graph $G' = (N', A')$ is a spanning subgraph of $G = (N, A)$ if $N' = N$ and $A' \subseteq A$. [cf. Ahuja et al. 1994, p. 2]

The minimum cost flow problem aims to ship commodities from all source nodes to all sink nodes such that the sum of costs, associated with using an arc, is minimized. The structure looks as follows:

Minimize

$$\sum_{(i,j) \in A} c_{ij} x_{ij} \quad (2.1a)$$

subject to

$$\sum_{\{j:(i,j) \in A\}} x_{ij} - \sum_{\{j:(i,j) \in A\}} x_{ij} = b_i, \quad \forall i \in N \quad (2.1b)$$

$$l_{ij} \leq x_{ij} \leq u_{ij}, \quad \forall (i, j) \in A \quad (2.1c)$$

x_{ij} corresponds to the quantity of commodities travelling along arc $(i, j) \in A$. Equation 2.1b enforces that the difference of incoming and outgoing flows equals the demand of the node. This equation is called the mass-balance equation [Ahuja et al. 1994, p. 3] or conservation-of-flow equation [Guisewite and Pardalos 1990, p. 75]. Equation 2.1c sets upper and lower bounds for x_{ij} and is called flow bound constraint. The most common minimum cost flow problem is the shortest path problem, in which $b_i = +1$ and $b_j = -1$. i corresponds to the start and j to the end node; weights of arcs are associated with their length. The shortest path problem is widely used because it is easy to solve efficiently and is often of interest in real-world problems in the private or public sector. [cf. Ahuja et al. 1994, pp. 5 sqq.]

2.1.2 Facility Location Problems in Operations Research

The importance of FLPs is reflected in the plethora of available papers.¹ Decisions of where to place new facilities are too important for not being the focus of mathematical optimization because they are costly and affect future operations. More often than not, mathematical models are utilized to incorporate as many parameters as possible in order to maximize the benefit associated with a facility location decision. The benefits of siting a facility can be manifold and differ between sectors and companies. Maximizing profit or minimizing costs, minimizing failure or maximizing service level could be the objective of new facilities, just to name a few. Because of this great variety it is necessary to define the FLP for this work more precisely.

FLPs in Operations Research can be categorized as follows: [cf. Melo et al. 2008, p. 402, cf Klose and Drexel 2003, p. 2, cf. ReVelle and Eiselt 2004, pp. 8 sqq., and Snyder 2006]

- single-product vs multi-product
- single-echelon vs multi-echelon
- one type of facility vs several types of facilities
- capacitated vs uncapacitated
- discrete vs continuous
- static vs dynamic
- single-period vs multi-period
- deterministic vs stochastic vs scenario-based

¹ Domschke and Drexel 1985 already comprises roughly 1500 references to location and layout models.[cf. ReVelle and Eiselt 2004, p. 1]

Other categorizations are conceivable, as well, but they do not help to clarify the context of the FLP in this work and are neglected. Following the sequence in which the different characteristics are mentioned, the FLP can be categorized as single-product, multi-echelon, regarding different types of facilities, capacitated, and discrete. Here, the only commodity considered is brine. It can be distributed either from production or storage facilities. The FLP is multi-echelon because of existing dependencies between storage and production facilities. Brine procurement from external sources is neglected and silos can only distribute the amount of brine they received from brine machines. All types of facilities are capacitated in terms of an upper capacity or an upper capacity per time unit. Moreover, the FLP is discrete because production and storage facilities can only be assigned to predefined locations in a network. Activating facilities at specific locations is expressed with binary variables, leading to a combinatorial optimization problem.

The remaining three characteristics need to be explained in more detail. In general, the FLP is considered static. After all, production and storage facilities shall be assigned to prospective locations once. Relocating facilities is not envisaged. However, in chapter 1 it is mentioned that a new concept is introduced. This concept comprises to locate a brine machine on a boat. This brine machine could travel along the coast so that both dynamic and static FLPs must be considered. Note that only the location of the boat is subject to change. Other aspects, like varying capacities or costs are not considered. The FLP in this work is designed as a multi-period model. As a consequence, it must be determined whether depreciation is considered or not. Supposing that a long temporal horizon is chosen, costs might be considered as net present value to compare investments at different dates, as demonstrated by Kostin et al. 2011. In this work, this is not required. The temporal horizon is short and investments are made up front. It is also

assumed that variable costs do not carry enough weight to make a difference to the FLP and are therefore neglected. Whether the FLP is deterministic, stochastic, or scenario-based depends on the quality of information upon which the facility location decision is based. In general, decisions can be made under certainty, risk, or uncertainty.[cf. Rosenhead et al. 1972, p. 415] The difference between risk and uncertainty is the linkage between decision and outcome. Under risk, information on the occurrence probability of an event is known or discrete scenarios exist. Under uncertainty, this information is not available and worst-case scenarios try to minimize damage. As described in section 1.2, the FLP considers weather as a source of variability. Therefore, weather records are utilized and occurrence probability of snowfall in an area is available. A decision is made based on risk so that both stochastic and scenario-based optimization could be used. Section 2.2 deals with the difference between those two approaches in depth and eventually defines the type of optimization used in this work.

Having defined the requirements for the FLP in this work, general FLP approaches and models are discussed next. FLPs can further be classified according to their objective function and their designated application, as presented below:

- P-Median Models
- P-Center Models
- Covering Models
- Competitive Models
- Flow Capturing Models
- Routing Location Models

- Multi-Objective Location Models

Probably the most commonly used models are the p-median and the p-center. In the p-median model, p facilities are sited so that the sum of travelled distances to satisfy all demand is minimized.[cf. Current et al. 1990, p. 297] In contrast, the p-center model determines locations for p centers such that the maximum distance between prospective facility locations and assigned demands is minimized.[cf. Klose and Drexl 2003, p. 4] The former is also called minisum or Weber problem, the latter is also called minimax or Rawls problem.[cf. Hansen et al. 1985, p. 1251] The objective of covering models is to maximize the total weight of covered demands, where a demand is considered covered when it occurs in less than a specified distance from the prospective facility location to which it is assigned.[Berman and Krass 2002 and Church and ReVelle 1974] Covering models can also be rephrased for the case in which the quantity of facilities, which is required to cover all demand, is to be minimized.[cf. Klose and Drexl 2003, p. 7] Competitive models aim at maximizing sales or market share in a competitive environment, in which at least two competing players try to expand their influence.[cf. Klose and Drexl 2003, p. 5] Hodgson 1990 presents another type of FLP. He regards demands as flows within a network and sites facilities according to their size. The objective is to capture as much flow as possible. In contrast to all models mentioned above, the remaining models, namely routing location and multi-objective location models, enhance the aforementioned ones and can be combined with all of them. Routing location models combine both siting of facilities and calculating delivery routes. Their formulation and solution is therefore extremely complicated.[cf. ReVelle and Eiselt 2004, p. 14] Aside from their impracticability for real-world problem sizes, other problems occur, too. For example, facility location and routing problems often work

with different temporal horizons and different degrees of abstraction so that it is difficult to combine them.[cf. Klose and Drexl 2003, p. 18 and Akca et al. 2008, p. 15] Multi-objective location models try to find the optimum based on several objectives. What sounds alluring is, in fact, often difficult to realize. It is not automatically guaranteed that an optimal solution can be found for a model with several objectives. Instead, the ultimate objective is rather difficult to find and results based on many objectives might lead to wrong conclusions.[cf. Klose and Drexl 2003, p. 18] An extensive review on multi-objective analysis for FLPs can be found in Current et al. 1990.

One goal stated in chapter 1 is to find a solution which expedites brine spreading operations. Therefore, facilities are placed in order to reduce the sum of travelled distances to meet demand. This objective coincides with the objective of the p-median model, which is chosen to address the problem. Static and dynamic FLPs can both be implemented. The p-median model is NP-complete, meaning that it cannot be solved efficiently, so that the set of prospective locations should be chosen subtly in order to decrease the size of the problem.[cf. ReVelle and Eiselt 2004, p. 7, cf. Current et al. 1990, p. 296] Hakimi's theorem, which holds true for minisum problems, is extremely advantageous in this context.[Hakimi 1965, Hakimi 1964, and cf. Snyder 2006, p. 5] It states that optimal solutions can only occur at nodes, due to concave distance functions. Therefore, it is sufficient to restrict the set of predefined locations to a subset of all nodes.

2.1.3 Facility Location Models

The p-median model is chosen to address the problem in this work. In this subsection, models, which are either related to snow removal operations or p-median models in general, are presented. Static FLPs are described first, dynamic FLPs are described afterwards.

In the elaborate review on winter road maintenance by Perriera, Langevina, et al. 2004, a simplified mixed integer model by Kandula and Wright 1997 is presented to integrate sector design, depot location, and fleet sizing for spreading operations. The objective function focuses on minimizing the sum of all distances of the shortest chains between road segment and associated depot. The underlying assumption is that trucks start and end their operation from the same depot.

Campbell and Langevin 1995 deal with a case study for the city of Montreal. Snow removal and disposal operations are explained and an integrated decision support is presented. Their objective function aims at minimizing transportation costs as well as variable and fixed costs for operating and establishing disposal sites. Although their paper deals with disposal sites instead of facilities, this does not make a difference to the model because in both cases, capacity restrictions are enforced and travel distances are considered.

Jang et al. 2010 discuss a situation in which the Missouri Department of Transportation (MoDOT) wants to reduce expenses for snow removal operations. To support their decision, a systematic, heuristic-based optimization approach is devised which simultaneously relocates depots, changes sector and vehicle route designs, determines a schedule for vehicles and reconfigures the available fleet. It becomes apparent that the focus of this paper is on the integration of several single optimization problems to overcome local optima. Their objective minimizes the quantity of spreading and plowing trucks while keeping the service level of operations, expressed by the cycle time for each road segment, at a constant level. In contrast to Campbell and Langevin 1995, depots are not placed based on the sum of distances to each road segment but on the accessibility to as many routes as possible.

Many dynamic FLPs can be found in Melo et al. 2003, pp. 4 sqq.

A new discrete FLP model is presented, too, in which operating costs of a supply chain network shall be minimized. Relocating facilities is only possible when its inventory has been depleted or has been shifted to another facility beforehand. Once closed, a facility cannot be reopened again and newly opened facilities must remain open to the end of the planning horizon.

In Klose and Drexel 2003, a dynamic version of the uncapacitated FLP is presented. In every period, facilities can be activated or deactivated, causing closing or opening costs. The model is formulated as a quadratic integer program and a linearization is provided, too. Their model also takes into account that when a status of a depot changes from closed to opened or vice versa it must keep that status for at least τ_{max} periods. Formally, this is described with $y_{t+\tau,j} \geq y_{tj}$ for $\tau = 1, \dots, \tau_{max}$. The model presented by van Roy and Erlenkotter 1982 restricts the opening and closing of facilities to only a few predefined periods in which transitions are allowed.[cf. Klose and Drexel 2003, p. 15]

2.2 Robust Optimization

Robust optimization (RO) is a part of Operations Research which is designed to find robust solutions to non-deterministic problems.[cf. Scholl 2001, p. 173] Stochastic and scenario-based approaches are available. The difference between those two is that scenario-based models find a solution based on discrete parameter combinations whereas non-deterministic parameters in stochastic models are described as continuous distribution functions. By introducing an index for different scenarios, previously stochastic variables are considered deterministic and the model can be solved with well known deterministic optimization techniques. This approach, in which variability is eliminated, is called indirect consideration of risk.[cf. Scholl 2001, p. 184]

The opposite approach, that is incorporating all available information as completely as possible in a stochastic optimization, is called direct consideration of risk.[cf. Scholl 2001, p. 196] Generally, the scenario approach is preferred.[cf. Scholl 2001, p. 184] Less information is required and models can be implemented faster. After all, devising scenarios is easier than obtaining probability distribution functions to parameters in real world problems. Another advantage of using scenarios is that the level of risk aversion can be determined in a more realistic manner, due to a smaller solution space. The difference between different solutions becomes more obvious and tendencies are better to notice. Especially for long-term projects with a lot of risk, these tendencies are very important. Because of these advantages, only the scenario approach is considered in this work.

In this section, general approaches of scenario-based models are described first before alternative configurations of these models are presented.

2.2.1 General Approaches

According to Scholl 2001, pp. 98 sqq., robustness can be subdivided into the following six criteria:

- result robustness,
- solution robustness,
- model robustness,
- information robustness,
- planning robustness, and
- evaluation robustness.

The criteria result-robust and solution-robust both describe solutions which are superior to others in all scenarios. The only difference is how the superiority is expressed. Using the criterion result-robust indicates that a solution always achieves the best value. In contrast, using the criterion solution-robust indicates that a solution always differs the least from the optimal value in a scenario. This gap is interpreted as opportunity loss or regret.[cf. Snyder 2006, p. 20] A solution is considered model-robust if the solution ensures validity in all scenarios, in contrast to other solutions. In case a model's underlying information is subject to change or only little information is considered, an information-robust solution is desirable, meaning that the solution is tolerant against not or insufficiently considered scenarios. Insufficiently considered scenarios often occur when the acquisition of further information is neglected, due to associated costs. A planning-robust solution might be advantageous when the solution is implemented in a subsequent manner, similar to the rolling horizon approach. The criteria applies to solutions which, after implementation, deviate the least from an initially envisaged solution. Finally, a solution is evaluation-robust when it is superior to others despite using different evaluation criteria. This robustness is required when evaluation criteria are diffuse and cannot be defined well. In robust optimization, most of the models are designed to meet one of the first three criteria; satisfying one of the second three criteria is more difficult because of a high degree of abstraction. They are only mentioned for the sake of completeness.

As described, superior solutions can be determined in two different ways, namely finding optimal values or minimizing regret. The associated model types are the following: [Scholl 2001, cf. and Snyder 2006]

- cost models, or

- regret models.

Cost models focus on costs, either on actual costs or penalty costs for shortcomings, and therefore aim for result- and model-robust solutions. Regret models determine solutions based on their opportunity loss, either of actual opportunity loss or artificial opportunity loss for shortcomings, across all scenarios and, thus, addresses solution and model robustness. Mulvey et al. 1995 do not distinguish between result and solution robustness and term both solution robustness.[cf. Snyder 2006, p. 31] As a consequence, cost and regret models satisfy the same robustness criteria. Snyder (2006) goes a step further and proves that these types are in fact equivalent because problems can be transferred from one type to the other. Therefore, available configurations described in the following subsection apply to both model types.[cf. Snyder 2006, p. 21] It is up to the modeler to opt between them. In this work, the regret model is chosen because its objective can be expressed either as an absolute or relative value. The downside of this approach is, however, that a model needs to be solved more often because an optimal value for each scenario must be determined beforehand.

2.2.2 Configuration of Approaches

The objective of regret models can either be to minimize regret or to minimize the maximum regret, referred to as minimax regret. Minimax regret models are chosen in situations in which a system failure should be avoided by all means or in the presence of uncertainty. This extremely risk averse approach is equivalent to a worst-case consideration where the worst case is overly emphasized and poor results may be obtained for other scenarios. For that reason, it is not recommended to use this approach for situations in which scenarios are categorized as "low/medium/high" because only the

most extreme scenario type is taken into account.[cf. Snyder 2006, p. 20] As will be explained in chapter 4, scenarios with a "low/medium/high" structure are used so that minimax regret models are neglected in this work.

Probably the first minimum regret model is devised by Mulvey et al. 1995. Their intention was to find a compromise between solution and model robustness and therefore split up the optimization model into a structure and control component.[cf. Scholl 2001, p. 174] In the structure component, variables are scenario-independent and constraints are strictly enforced; in the control component, variables are scenario-dependent and slack variables are introduced to relax constraints. Their model looks as follows:[Scholl 2001, p. 175]

Minimize

$$\varphi(z) + w * \lambda(v_1, \dots, v_K) \quad (2.2a)$$

subject to

$$Ax = b \quad (2.2b)$$

$$z_k = c^T x + d_k^T y_k \quad \forall k \in K \quad (2.2c)$$

$$B_k x + C_k y_k + v_k = e_k \quad \forall k \in K \quad (2.2d)$$

$$y_k, x \leq 0, v_k \text{ unrestricted} \quad \forall k \in K \quad (2.2e)$$

Equation 2.2a is the weighted sum of two functions. $\varphi(z)$ corresponds to the initial objective function and ensures solution robustness, $\lambda(v_1, \dots, v_K)$ corresponds to the violation of control constraints and ensures model robustness. The weighing factor w is a predefined parameter defining the level of risk aversion. Several optimization runs with different values for w are recommended to obtain a meaningful interpretation of its effect on the solu-

tion.[cf. Scholl 2001, p. 177] The objective function determines how to obtain a solution based on the objective values and violations across all scenarios. For example, the objective of the model could be to minimize the expected regret by replacing $\varphi(z)$ with $\sum_k p_k * z_k$, where k corresponds to the scenarios and p to the probability that scenario k occurs. Other configurations are conceivable, as well.[cf. Scholl 2001, p. 176] It is noteworthy that robust optimization models are large by their nature and non-linear objective functions should be avoided. Equation 2.2b is the structure component of the model with a deterministic variable x ; equation 2.2c corresponds to the objective function for each scenario. The control component of the model is represented in equation 2.2d. y_k are non-deterministic variables associated with the scenarios and v_k are slack variables. A , B , C , c , and d are parameters. Scholl 2001 concludes that the application of this model is prudent for only a few specific cases. Major shortcomings are the derivation and interpretation of the weighing factor w and the assumption that uncertainty cannot occur in the structure component of the model. When it comes down to real world problems, this assumption is weak. On the contrary, this model incorporates crucial approaches for dealing both with solution and model robustness – two important characteristics for robust optimization.[cf. Scholl 2001, p. 181]

Chapter Three

COST-BENEFIT OPTIMIZATION

The goal of the FLP in this work is to site facilities in order to expedite spreading operations by increasing the availability of brine. In contrast to other papers related to snow removal, it is not desired to identify a central point within a network since the underlying assumption in this work is that the facilities to be sited only serve as replenishment points. Trucks start and end their operations from already existing, probably centralized depots. By increasing the availability of brine, spreading trucks have more replenishment opportunities and time for replenishment dwindles. Optimally, a replenishment point is nearby whenever a truck runs out of brine. This, however, is difficult to realize since truck routes are contingent on weather, which in turn is subject to change. For this reason it is assumed that no predefined routes exist. Instead, spreading operations are simplified such that brine is delivered from any brine storage or production facility to each road segment *individually*, using the shortest path. Trucks are completely neglected and their routes are irrelevant for the FLP.

The benefit of setting up a facility at a distinct location is the sum of distances to deliver brine to all road segments. However, by entirely focusing on distances between facilities and road segments, all roads are considered equal and facilities would rather be located close to clusterings of small roads, i.e. neighborhoods, than to arterial roads or highways. To take into account that road segments of highways are longer and that they consist of several

lanes, a decision is obtained based on *consumption miles*. This artificial unit is the distance to a road segment weighted with its demand. The smaller the associated sum of consumption miles of a facility location alternative, the more beneficial it is. Less time is spent on the road and operations are expedited. Unlike Kandula and Wright 1997, p. 162 who use distance chains between road segments and silos, it is only important how to get brine from a silo to a road segment. Dead mileage cannot be considered since trucks do not have to return to the same replenishment point.

In section 3.1, the static multi-facility type location model is introduced, dealing with a brine network consisting of brine machines, silos, and road segments. In the following section 3.2, the concept of a mobile brine machine is added to this base model.

3.1 Static Multi-Facility Type Location Model

3.1.1 Premises and Assumptions

The base model is a static FLP in which investments are made up front. It can either be invested in brine machines or silos, whereas the storage capacity of silos may be augmented. Opening new or closing old facilities as well as resizing storage capacities of silos in later periods is not allowed. The premise of this model is a street network in which prospective brine machine and silo locations are known. The facility location model is therefore discrete and only those locations are considered. Road segments can only be delivered from silos, which in turn are replenished by brine machines. The connection between brine machines and road segments is neglected in order to reduce the size of the model. Shifting brine between silos is not intended.

The street network is subdivided into the following three road types:

- non-maintained

- maintained (primary)
- maintained (secondary)

Non-maintained roads are irrelevant for the facility type location problem since no brine demand occurs. However, they might be important for establishing the shortest path between silo and road. In the presence of brine shortage, it is important to set priorities between different types of roads. Highways and arterial roads are major traffic pathways and therefore must be served entirely. These are considered primary roads. In contrast, the preparation of medium and small volume roads is less important. A minimum service level is imposed so that a slack is allowed for those secondary roads.

Referring to the approach in which routes are neglected and demand of road segments is met individually, the street network needs to be modified. Double directed roads are split in two single-directed flows, according to the procedure explained in section 2.1.1, and their brine demand is based on their length times the number of lanes the associated road segment contains. A major problem of this arc splitting is that the size of the model increases tremendously, making it computationally more challenging to obtain a solution. Therefore, it is reasonable to apply preprocessing techniques, meaning that instead of filling the model with all available information, only required information is passed on to it. ReVelle and Eiselt 2004, p. 2 suggest to calculate shortest paths between silos and the starting point of flows beforehand, typically with a $O(n^3)$ based Dijkstra algorithm. It can also be calculated with a minimum cost linear program. The idea is the same, namely that the calculated distances are passed on to the model as parameters. In the following, always two distances are calculated, one without travelling along a flow's end point and one with travelling along it. The idea behind this

additional calculation is that trucks, although neglected in this model, will eventually distribute brine. Calculating distances without travelling along a flow's endpoint ensures that trucks do not use the road they are supposed to prepare in the opposite direction and u-turns are avoided. However, not using the endpoint can either make it impossible to reach the start point of a flow or results in major detours. The former often occurs at a network's boundary. In these cases, it is assumed that trucks take u-turns.

3.1.2 The Base-Model

Sets

$b \in \mathcal{B}$	$:= \{1, \dots, B\}$	Prospective brine machine locations
$i \in \mathcal{I}$	$:= \{1, \dots, I\}$	Primary flows
$j \in \mathcal{J}$	$:= \{1, \dots, J\}$	Secondary flows
$p \in \mathcal{P}$	$:= \{1, \dots, P\}$	Periods
$s \in \mathcal{S}$	$:= \{1, \dots, S\}$	Prospective silo locations
$t \in \mathcal{T}$	$:= \{1, \dots, T\}$	Silo size add-ons

Parameters

– Basic –

$budget$	Initial budget for investments
$BigM$	Large number
$MinServiceLevel$	Minimum accepted service level of secondary flows

– Brine machines –

$bPCAPA$	Production capacity
$bACOST$	Activation costs (set up costs)
$DIST_{bs}$	Distance between b and s

– Silos –

$sBaseSCAPA$	Base storage capacity
$AddSCAPA_t$	Additional storage capacity associated with t
$sACOST$	Activation costs
$ACOST_t$	Activation costs of t
$DIST_{s(i/j)}$	Distance between s and i/j
– <i>Roads</i> –	
$DEM_{(i/j)p}$	Brine demand of i/j in p

Variables

$\alpha_{b/s/st}$	$\alpha \in \mathbb{B}$	Activation of $b/s/st$
δ_{sp}	$\delta \in \mathbb{R}^+$	Storage level of s in p
ε_{jp}	$\varepsilon \in \mathbb{R}^+$	Unmet demand of j in p
ρ_{bp}	$\rho \in \mathbb{R}^+$	Brine production of b in p
$\chi_{(bs/si/sj)p}$	$\chi \in \mathbb{R}^+$	Brine delivered $(b \Rightarrow s/s \Rightarrow i/s \Rightarrow j)$ in p

The objective is to minimize the sum of all consumption miles between brine machines and silos as well as between silos and both primary and secondary flows in all periods.

$$\sum_{p \in \mathcal{P}} \sum_{s \in \mathcal{S}} \left[\sum_{b \in \mathcal{B}} \chi_{bsp} * DIST_{bs} + \sum_{i \in \mathcal{I}} \chi_{sip} * DIST_{si} + \sum_{j \in \mathcal{J}} \chi_{sjp} * DIST_{sj} \right] \quad (3.1a)$$

The multi-facility type location problem is determined subject to the follow-

ing constraints:

$$\sum_{p \in \mathcal{P}} \rho_{bp} \leq \alpha_b * BigM \quad \forall b \in \mathcal{B} \quad (3.1b)$$

$$\rho_{bp} \leq bPCAPA \quad \forall p \in \mathcal{P}, b \in \mathcal{B} \quad (3.1c)$$

$$\sum_{s \in \mathcal{S}} \chi_{bsp} = \rho_{bp} \quad \forall p \in \mathcal{P}, b \in \mathcal{B} \quad (3.1d)$$

Constraints 3.1b and 3.1c ensure that a brine machine can only produce brine when activated and that it cannot produce more than its production capacity allows. Brine machines cannot store brine and all produced brine must be delivered to silos. (3.1d)

$$\sum_{p \in \mathcal{P}} \left[\sum_{i \in \mathcal{I}} \chi_{sip} + \sum_{j \in \mathcal{J}} \chi_{sjp} \right] \leq \alpha_s * BigM \quad \forall s \in \mathcal{S} \quad (3.1e)$$

$$\begin{aligned} \delta_{sp} &= \delta_{(s,p-1)} + \sum_{b \in \mathcal{B}} \chi_{bsp} - \\ &\quad \sum_{i \in \mathcal{I}} \chi_{sip} - \sum_{j \in \mathcal{J}} \chi_{sjp} \quad \forall p \in \mathcal{P}, s \in \mathcal{S} \end{aligned} \quad (3.1f)$$

$$\begin{aligned} \delta_{sp} &\leq sBaseSCAPA + \\ &\quad \sum_{t \in \mathcal{T}} \alpha_{st} * AddSCAPA_t \quad \forall p \in \mathcal{P}, s \in \mathcal{S} \end{aligned} \quad (3.1g)$$

$$\sum_{t \in \mathcal{T}} \alpha_{st} \leq 1 \quad \forall s \in \mathcal{S} \quad (3.1h)$$

The silo constraints comprise the activation of a silo (3.1e), its massbalance (3.1f) and the activation of additional storage capacity (3.1g and 3.1h). Silos are modelled as hubs with storage capacity, although they must be activated in order to pass brine to flows. This approach was chosen because brine machines can only deliver brine to flows via silos. Consequently, the amount of brine distributed from a silo may very well exceed its storage capacity. The massbalance constraint is self-explanatory. It is noteworthy that $\delta_{s,0}$, that is the initial storage level of a silo, is filled to maximum capacity. Later,

the model undergoes a stress test with subsequent snow storms and it is assumed that before this sequence strikes, maximum capacity is available. The implementation of the stress test is explained in chapter 4. Constraint 3.1g activates additional capacity and constraint 3.1h limits the quantity of selected add-ons to one.

$$\sum_{s \in \mathcal{S}} \chi_{sip} = DEM_{ip} \quad \forall p \in \mathcal{P}, i \in \mathcal{I} \quad (3.1i)$$

$$\sum_{s \in \mathcal{S}} \chi_{sjp} + \varepsilon_{jp} = DEM_{jp} \quad \forall p \in \mathcal{P}, j \in \mathcal{J} \quad (3.1j)$$

$$\sum_{j \in \mathcal{J}} \varepsilon_{jp} \leq MinServiceLevel * \sum_{j \in \mathcal{J}} DEM_{jp} \quad \forall p \in \mathcal{P} \quad (3.1k)$$

Constraints 3.1i and 3.1j are the demand constraints. As described earlier, primary flows must be served entirely whereas secondary roads must only be served up to a specific percentage. Constraint 3.1k imposes an upper limit for unmet demand.

$$\begin{aligned} & \sum_{b \in \mathcal{B}} \alpha_b * bACOST + \\ & \sum_{s \in \mathcal{S}} \left[\alpha_s * sACOST + \sum_{t \in \mathcal{T}} \alpha_{st} * sAddCOST_t \right] \\ & \leq budget \end{aligned} \quad (3.1l)$$

Finally, constraint 3.1l restricts available investments.

3.2 Dynamic Facility Location Problem

3.2.1 Development of the Concept "Water Alternative"

Brine spreading operations are expensive because both salt and water are required in huge amounts. To reduce these costs, the idea is to use natural salt water to produce brine. The water supply is free of charge and less salt is

required since a high salinity level already exists. For instance, the Atlantic has a salinity level of approximately 3.5%.[Fuglister 1960, cf.] Starting from this basic idea, a concept is evaluated in this work in which a brine machine is located on a boat. The boat could provide a mobile support which travels along the coast and replenishes trucks at harbours or alternative access points. In contrast to the static FLP in section 3.1, a replenishment point could indeed be nearby whenever a truck runs out of brine.

The following aspects have an influence on the success of this concept:

1. Maintained road segments are close to the coastline.
2. Access points are plenty in numbers and evenly distributed.
3. Routes of spreading trucks and routes of boats are synchronized.

The closer all maintained roads are to the coastline and the more access points exist, the more dead mileage can be reduced. Spreading routes can be planned in chains, starting and ending their operations at the coast. Furthermore, well coordinated operations could even increase the potential of this concept by avoiding trucks to wait for replenishment. Although reduced production costs and the mobility aspect seem desirable, using a boat can induce additional costs for rent or purchase, fuel, and other aspects such as insurance. This evaluation addresses the question of whether a mobile silo is a serious alternative.

The concept of a mobile brine machine is analysed only for the period that the aforementioned stress test lasts. A long-term analysis is not covered in this work and costs for maintenance and storage during summer are not explicitly considered. These costs as well as operational costs for fuel are implicitly considered by higher investment costs. The activity of a boat

is not apparent, due to neglected truck routes, and estimating fuel costs is difficult. Taking all of these costs into consideration, the concept seems quite unappealing so far. But setting up stationary brine machines also leads to major consequential costs for wiring and plumbing. Conversely, these costs are comparatively low for setting up a brine machine on a boat. Water can be directly pumped out of the ocean and energy is provided by the boat's engine. The evaluation of this concept will eventually conclude in a sensitivity analysis which determines an upper investment cost up to which this concept is superior to conventional silos and brine machines.

In the following, this concept is called the *water alternative*. Since the idea is daring enough, the water alternative only comprises one boat in which both brine production and storage is centralized. Assuming that several access points exist, it cannot be ensured that brine can be delivered to all these points at once; a need-based deployment is not possible either due to a lack of truck routes. Therefore, these access points are grouped into *water stations*. Whenever the boat is at a water station, all associated access points can be used to feed the street network with brine. It is apparent that only the closest access point to either a flow or a silo is utilized in order to minimize consumption miles.

Reducing the problem size by preprocessing distances is even more important in this context. In order to determine the shortest path between a water station and, for instance, a flow, the shortest distance between every access point of this water station and the flow is required. Likewise to the preprocessing of shortest distances in section 3.1, each distance is calculated twice. The computational burden is significant, as it will be shown in chapter 5, but it eventually helps to reduce the problem size by reducing the amount of variables.

In the following subsection 3.2.2, the base model from section 3.1

will be augmented by the new concept to a dynamic multi-echelon and multi-facility type location model.

3.2.2 Model Extension I

Additional sets

$a \in \mathcal{A}$	$:= 1$	Water alternative
$m \in \mathcal{M}_w$	$:= \{1, \dots, M_w\}$	Mobile silos associated with w
$w \in \mathcal{W}$	$:= \{1, \dots, W\}$	Water stations

Additional parameters

– *Mobile silos* –

$DIST_{m(s/i/j)}$	Distance between m and $s/i/j$
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– *Water alternative* –

$aACOST$	Activation costs
$aSCAPA$	Storage capacity
$aPCAPA$	Production capacity

– *Water stations* –

$DIST_{w(s/i/j)}$	Distance between w and $s/i/j$
$DIST_{w(s/i/j)} := \min\{DIST_{m(s/i/j)} \mid \forall m \in \mathcal{M}_w\}$	

Additional variables

α_a	$\alpha \in \mathbb{B}$	Activation of a
α_{wp}	$\alpha \in \mathbb{B}$	Activation of w in p
δ_{ap}	$\delta \in \mathbb{R}^+$	Storage level of a in p
ρ_{ap}	$\rho \in \mathbb{R}^+$	Brine production of a in p
$\chi_{(aw/ws/wi/wj)p}$	$\chi \in \mathbb{R}^+$	Brine delivered $(a \Rightarrow w/w \Rightarrow s/w \Rightarrow i/w \Rightarrow j)$ in p

The objective still is to minimize the sum of all consumption miles over all

periods.

Eq.3.1a+

$$\sum_{p \in \mathcal{P}} \sum_{w \in \mathcal{W}} \left[\sum_{s \in \mathcal{S}} \chi_{wsp} * DIST_{ws} + \sum_{i \in \mathcal{I}} \chi_{wip} * DIST_{wi} + \sum_{j \in \mathcal{J}} \chi_{wjp} * DIST_{wj} \right] \quad (3.2a)$$

In addition to constraints 3.1b, 3.1c, 3.1d, 3.1e, 3.1g, 3.1h, and 3.1k, the following constraints apply:

$$\begin{aligned} \delta_{sp} &= \delta_{(s,p-1)} + \sum_{b \in \mathcal{B}} \chi_{bsp} + \sum_{w \in \mathcal{W}} \chi_{wsp} - \\ &\quad \sum_{i \in \mathcal{I}} \chi_{sip} - \sum_{j \in \mathcal{J}} \chi_{sjp} \quad \forall p \in \mathcal{P}, s \in \mathcal{S} \end{aligned} \quad (3.2b)$$

The massbalance constraint for silos is augmented such that replenishment from water stations is allowed, as well.

$$\sum_{p \in \mathcal{P}} \sum_{w \in \mathcal{W}} \chi_{awp} \leq \alpha_a * BigM \quad \forall a \in \mathcal{A} \quad (3.2c)$$

$$\delta_{ap} = \delta_{(a,p-1)} + \rho_{ap} - \sum_{w \in \mathcal{W}} \chi_{awp} \quad \forall p \in \mathcal{P}, a \in \mathcal{A} \quad (3.2d)$$

$$\delta_{ap} \leq aSCAPA \quad \forall p \in \mathcal{P}, a \in \mathcal{A} \quad (3.2e)$$

$$\rho_{ap} \leq aPCAPA \quad \forall p \in \mathcal{P}, a \in \mathcal{A} \quad (3.2f)$$

Constraints 3.2c, 3.2d, 3.2e, and 3.2f activate the water alternative and manage its brine production, its brine storage, and its delivery to water stations. The water alternative is activated when one of the water stations is utilized. It is equatable with renting or purchasing a boat. Naturally, brine production and storage capacity are limited, whereas storage capacity cannot be augmented due to limited space and buoyancy. Likewise silos, the storage

capacity of the water alternative is initially filled to capacity.

$$\sum_{s \in \mathcal{S}} \chi_{wsp} + \sum_{i \in \mathcal{I}} \chi_{wip} + \sum_{j \in \mathcal{J}} \chi_{wjp} \leq \alpha_{wp} * BigM$$

$$\forall p \in \mathcal{P}, w \in \mathcal{W} \quad (3.2g)$$

$$\sum_{a \in \mathcal{A}} \chi_{awp} = \sum_{s \in \mathcal{S}} \chi_{wsp} + \sum_{i \in \mathcal{I}} \chi_{wip} + \sum_{j \in \mathcal{J}} \chi_{wjp}$$

$$\forall p \in \mathcal{P}, w \in \mathcal{W} \quad (3.2h)$$

$$\sum_{w \in \mathcal{W}} \alpha_{wp} \leq 1 \quad \forall p \in \mathcal{P} \quad (3.2i)$$

Water stations can only pass brine to silos or flows when they are activated, meaning that the boat is located at a specific water station in period p (3.2g). They are not allowed to store anything so that all brine received from the water alternative must be passed on (3.2h). This constraint is important to centralize brine production and storage of the water alternative. Since only one boat is considered, only one water station can be activated at a time (3.2i).

$$\sum_{s \in \mathcal{S}} \chi_{sip} + \sum_{w \in \mathcal{W}} \chi_{wip} = DEM_{ip} \quad \forall p \in \mathcal{P}, i \in \mathcal{I} \quad (3.2j)$$

$$\sum_{s \in \mathcal{S}} \chi_{sjp} + \sum_{w \in \mathcal{W}} \chi_{wjp} + \varepsilon_{jp} = DEM_{jp} \quad \forall p \in \mathcal{P}, j \in \mathcal{J} \quad (3.2k)$$

Constraints 3.2j and 3.2k refer to the demand constraints and are updated with the amount of brine received from water stations.

$$\sum_{a \in \mathcal{A}} \alpha_a * aACOST + \sum_{b \in \mathcal{B}} \alpha_b * bACOST +$$

$$\sum_{s \in \mathcal{S}} \left[\alpha_s * sACOST + \sum_{t \in \mathcal{T}} \alpha_{st} * sAddCOST_t \right]$$

$$\leq budget \quad (3.2l)$$

Budget constraint 3.2l considers investments in the water alternative.

Chapter Four

ROBUST EXTENSION

So far, a facility location model is presented, determining an optimal facility mix and associated locations based on deterministic data. Bearing in mind that both brine production and storage facilities are *only* utilized during snow storms, it is apparent that the FLP cannot entirely neglect variability. Both the locations of facilities and the choice of facility type are influenced by the weather. The location of a facility is likely to move towards areas which are rather prone to frequent snow storms; the facility type may change to the effect that more production facilities or larger silos are chosen. The change of facility types addresses the likelihood of brine shortage in the system – the second premise in chapter 1.

In this chapter, model 3.2 is extended by a non-deterministic component. Therefore, the indirect consideration of risk is used to take the aforementioned aspects into account.

4.1 Weather Scenarios

Risk can either be classified as systematic or unsystematic.[cf. Scholl 2001, p. 213] In the former case, factors are known which shape the outcome of a variable system although the probability of occurrence is unknown. In the latter case, it is assumed that such factors do not exist or that their effect on the outcome is not clear. Weather is a systematic source of variability which is based, among other factors, on geographical conditions and, thus,

follows specific patterns. For instance, the probability of snowfall nearby the coast is less than inland. Scholl 2001 suggests for these cases to use bygone system states, either in a direct or extrapolated form, to create scenarios. This would ensure that most of the possible states are considered and that a proper distribution of their occurrences can be obtained, as well. In the following, weather records are utilized to extend the deterministic model by a variable aspect.

A solution to the FLP is obtained by stressing the system. For this purpose, areas with similar weather patterns are identified. For each of these areas, differently intense weather scenarios are created, whereas the intensity is expressed by the number of inter-storm days. Keeping the planning horizon at the same level, a snow storm is assumed to hit every time the number of inter-storm days elapse. With fewer days between storms, more demand occurs and less time remains for refilling silos. The system is more prone to fail. The FLP is then solved for all intensity level - area combinations, whereas each of these combinations is considered equally likely. The following subsections elaborate this approach.

4.1.1 Consideration of Regional Differences

Information on snowfall is used to create scenarios in which flows have different demand patterns. These patterns are obtained by assigning flows to nearby weather stations. Contingent to the quantity of weather stations and the size of the street network, this assignment problem becomes difficult to solve; especially, when the assignment is required to be on a daily basis because records of weather stations are partially incomplete and no information is available for some periods. Unless alternative weather stations with valid information can be found within a given perimeter, it is assumed that it did not snow on that days. To reduce the size of the assignment model, a

regional decomposition is applied by adding a grid to a given street network as a second layer. The weather pattern for a group of flows is then represented by a single grid point and only each grid point and not each flow must be assigned to a weather station. Aside from a regional decomposition, a temporal decomposition is applied, as well. Each winter is considered as a scenario which is independent of other scenarios so that the following assignment problem can be solved scenario-wise.

Sets

$g \in \mathcal{G}$	$:= \{1, \dots, G\}$	Grid points
$n \in \mathcal{N}$	$:= \{1, \dots, N\}$	Weather stations
$p \in \mathcal{P}$	$:= \{1, \dots, P\}$	Periods

Parameters

$DIST_{gn}$	Distance (beeline) between g and n
$MaxDIST$	Maximum distance between g and n
$valid_{np}$	Valid weather information for n in p
$valid_{np} =$	$\begin{cases} 1 & \text{if measured snow height} \geq 0 \\ 0 & \text{otherwise} \end{cases}$

Variables

κ_{gnp}	$\kappa \in \mathbb{B}$	Assignment of g to n in p
ς_{gp}	$\varsigma \in \mathbb{B}$	Slack for g in p

Minimize

$$\sum_{p \in \mathcal{P}} \sum_{g \in \mathcal{G}} \left[\sum_{n \in \mathcal{N}} \kappa_{gnp} * DIST_{gn} + \varsigma_{gp} * MaxDIST \right] \quad (4.1a)$$

subject to

$$\sum_{n \in \mathcal{N}} \kappa_{gnp} + \varsigma_{gp} = 1 \quad \forall g \in \mathcal{G}, p \in \mathcal{P} \quad (4.1b)$$

$$\kappa_{gnp} \leq valid_{np} \quad \forall g \in \mathcal{G}, n \in \mathcal{N}, p \in \mathcal{P} \quad (4.1c)$$

The objective function 4.1a minimizes the accumulated distances between grid points and weather stations. The closer a weather station is to a grid point, the more accurately the recorded weather represents the actual weather. Whenever there is no weather station in the perimeter of p , a slack variable is activated and it is assumed that it did not snow. For every period in every scenario, either a weather station or a slack variable must be chosen (eq. 4.1b). Constraint 4.1c ensures that only those weather stations with valid information can be chosen.

Having obtained weather scenarios for each grid point, regional differences can be incorporated into the FLP. However, depending on the quantity of grid points, there might be quite a few scenarios to consider in a robust optimization problem and it is recommended to group grid points in order to reduce the size of the model.[Scholl 2001, p. 219] This grouping is based on the quantity of days where two grid points share exactly the same weather events; a weather event can only be snowfall or no snowfall. For analysing anti-icing measures, this information reduction is valid since the amount of brine sprayed on the road does not depend on the amount of snowfall or the type of snow. The clustering problem is formulated as a k-means problem in which the quantity of clusters is given by parameter k .

Parameters

$events_{g_1g_2}$	Unequal weather events between g_1 and g_2
$clusters$	Defined quantity of clusters

Variables

α_g	$\alpha \in \mathbb{B}$	Activating g as center
$\kappa_{g_1g_2}$	$\kappa \in \mathbb{B}$	Assignment of g_1 to g_2

Minimize

$$\sum_{g_1, g_2 \in \mathcal{G}} \kappa_{g_1g_2} * events_{g_1g_2}^2 \quad (4.2a)$$

subject to

$$\sum_{g \in \mathcal{G}} \alpha_g = clusters \quad \forall g \in \mathcal{G} \quad (4.2b)$$

$$\sum_{g_2 \in \mathcal{G}} \kappa_{g_1g_2} \leq \alpha_{g_1} * G \quad \forall g_1 \in \mathcal{G} \quad (4.2c)$$

$$\sum_{g_1 \in \mathcal{G}} \kappa_{g_1g_2} = 1 \quad \forall g_2 \in \mathcal{G} \quad (4.2d)$$

The objective is to minimize the sum of squares of different weather events of all grid points in a group (4.2a). Constraint 4.2b sets the quantity of considered centers, constraint 4.2c activates a grid point as a center and constraint 4.2d ensures that each grid point is part of a cluster.

4.1.2 Consideration of Brine Shortage

The street network is now divided into areas with similar weather patterns. The next step is to create different intensity levels for each of them to stress the system. The underlying assumption of this approach is that when a system is designed to withstand extreme scenarios, it ensures high service levels for other scenarios, as well.

As mentioned in the introductory paragraph of this chapter, inter-

storm days and the quantity of subsequently striking snow storms causes problems. The former is addressed by analysing the weather patterns of each center grid point determined above, whose records are assumed to be representative for the associated group. The latter is indirectly considered by choosing a planning horizon and simulating snow storms in an area every time the associated inter-storm days elapse. The amount of brine is not contingent to the amount of snow and it is of interest to see whether a system, whose silos are initially filled to capacity, could satisfy all constraints of the model over the entire planning horizon.

The intensity level is described with the probability that the chosen quantity of inter-storm days is less than the inter-storm days measured for an area. Therefore, a discrete frequency distribution of inter-storm days is required, where only those quantities are considered which are more than one and less than a predefined upper limit. The lower bound is set because salt is spread instead of brine during consecutive snow storms. The asphalt is wet and the salt sticks to it. Consequently, the facility mix must not be able to provide enough brine for consecutive days of operations without at least a day in between. The upper limit corresponds to the time to restore the initial situation, meaning to fill all silos to capacity. Referring to the aforementioned assumption that it is sufficient to focus on extreme scenarios, the burden of consecutively hitting snow storms is interrupted when the initial situation is restored so that inter-storm days beyond the upper limit can be neglected. Eventually, the robust optimization model is fed with the downward adjusted mean quantity of inter-storm days for each intensity level across all recorded winters.

At this point, it is important to address the aspect of computational effort. The scenarios for the robust optimization model are all possible intensity level - area combinations so that the number of scenarios is calculated

as the quantity of intensity levels to the power of the quantity of centers. Fewer centers are consequently preferred to avoid an exponentially increasing model size. However, when only a few centers are considered, regional differences become less obvious. Helpful in this context is that, due to the nature of integers, the set of inter-storm days is limited so that center - intensity level combinations occur more than once. As a consequence, it is reasonable to solve the robust optimization model only for unique combinations, which tremendously reduces the model size. To maintain the quality of information after compressing, the objective values of each scenario are weighted with their frequency of occurrence in the scenario-independent objective function. This compressing of information is only valid since the scenarios themselves are assumed to be equally likely. They are artificial weather - area combinations which are based on real data but for which no probability of occurrences are known.

4.2 Implementation

In section 2.2, it is explained why a regret model is chosen to address the problem in this work. The objective function can be expressed as a percentage of the optimal value and the quality of the solution becomes more obvious. Consumption miles is an idea not everyone is familiar with and the interpretation of the solution is difficult without any comparison. It is also explained in this section why the model must be formulated as a minimization problem. A minimax problem formulation would only focus on the worst case scenario and all other scenarios would be superfluous.

In the following subsections, the robust optimization model is devised and its formulation is given.

4.2.1 *Model structure*

The model presented in this work is based on the model by Mulvey et al. 1995, presented in section 2.2. This model is chosen as a starting point because it addresses two crucial aspects for non-deterministic optimization models, namely model and solution robustness. Model robustness means that the model is feasible so that the chosen facility type mix can serve all primary flows and can maintain the specified service level for secondary flows. Solution robustness means that facilities are located such that the expected regret is minimized.

Their model is divided into a structure and a control component. In the former component, parameters are deterministic, variables are scenario-independent, and constraints are strictly enforced. In contrast, the latter component addresses variability and constraints are relaxed by introducing slack variables, which are unbounded and are added to the objective function with a weighing factor w . This weighing factor is criticised because it cannot be determined that easily and the result of the model is difficult to interpret. Another point of criticism is the assumption that variability does only occur in the control component. Especially long-term decisions inhibit a great deal of variability and it is said to be weak to assume that no variability occurs in the structure component.[cf. Scholl 2001, p. 181] Taking the former point of criticism into consideration, the model in this work refrains from using a weighing factor and slack variables do not appear in the objective function. Instead, slack variables are restricted by constraint 3.1k, which imposes the minimum service level for secondary flows. Consequently, the model can be infeasible and model robustness is not ensured by the model. A sensitivity analysis is required to find the minimum service level for secondary flows for which the model is hardly feasible across all scenarios. The model

presented below only addresses solution robustness directly. The concerns that variability can also appear in the structure component are neglected because they do not apply the model in this work. Investments are made up front so that deterministic data can be assumed and the only remaining source of variability is weather, which only affects demand in the control component.

4.2.2 Model Extension II

Additional sets

$$k \in \mathcal{K} \quad := \{1, \dots, K\} \quad \text{Unique scenarios}$$

Additional parameters

$$p_k \quad \text{Frequency of occurrence of } k$$

$$z_k^* \quad \text{Optimal value for } k$$

Additional variables

$$\zeta_k \quad \zeta \in \mathbb{R}^+ \quad \text{Objective value for } k$$

The scenario-independent objective is to minimize the regret associated with a facility location decision (4.3a). The relative deviation from the optimal value in each unique scenario is weighted with the occurrence frequency of that scenario.

$$\sum_{k \in \mathcal{K}} p_k * \frac{\zeta_k - z_k^*}{z_k^*} \quad (4.3a)$$

A solution is obtained based on all constraints listed in model 3.2 and the following constraint 4.3b, which determines the scenario-dependent objective value for a facility location decision. As in model 3.2, these scenario-

dependent objective values are only based on consumption miles.

$$\begin{aligned}
& \sum_{p \in \mathcal{P}} \left[\sum_{s \in \mathcal{S}} \left[\sum_{b \in \mathcal{B}} \chi_{bskp} * DIST_{bs} + \right. \right. \\
& \quad \left. \sum_{i \in \mathcal{I}} \chi_{sikp} * DIST_{si} + \sum_{j \in \mathcal{J}} \chi_{sjkp} * DIST_{sj} \right] + \\
& \quad \sum_{w \in \mathcal{W}} \left[\sum_{s \in \mathcal{S}} \chi_{wskp} * DIST_{ws} + \right. \\
& \quad \left. \sum_{i \in \mathcal{I}} \chi_{wikp} * DIST_{wi} + \sum_{j \in \mathcal{J}} \chi_{wjkp} * DIST_{wj} \right] \Big] \\
& = \zeta_k \quad \forall k \in \mathcal{K}
\end{aligned} \tag{4.3b}$$

In constraint 4.3b, variables for brine transport, χ , are augmented by index k . The same applies to variables for brine production, ρ , brine storage, δ , unmet demand, ε , and the activation of water stations, α_w , since their values are scenario-dependent. In contrast, the activation of facilities, additional storage capacities, or the water alternative where a brine machine is located on a boat[cf.], α , are scenario-independent decisions. Constraints for scenario-independent variables sum up scenario-dependent variables across all scenarios; constraints for scenario-dependent variables are enforced scenario-wise.

Chapter Five

CASE STUDY: RHODE ISLAND TANK FARMS

This chapter presents the application of the developed models to a real world problem, i.e. the brine distribution problem in the State of Rhode Island in the United States of America.

5.1 Introduction

With approximately 1,500 square miles, Rhode Island is the smallest of the 50 states. Its climate is considered humid continental with hot summers and chilly winters. The average annual snowfall is 36.7 inches, 60% of which falls in December and January.

The Rhode Island Department of Transportation (RIDOT) is responsible for winter road maintenance for 37.8% of Rhode Island's street network, which consists of 8200 lane miles. Using only one brine machine and two silos at the same location in the center of Rhode Island so far, RIDOT wants to invest \$100,000 to establish brine operations as a first response to all types of snowfalls. The brine machine has a production capacity of 20,000 gals. per day and the two silos have a storage capacity of 5,000 gals. each. The initial situation is depicted in figure 5.1, in which prospective and existing brine machine and silo locations are tagged. This figure also shows a possible implementation of a water alternative. As it becomes apparent, the concept of distributing brine via water is indeed an alternative for Rhode Island. 17% of Rhode Island is covered with water and the shore can be

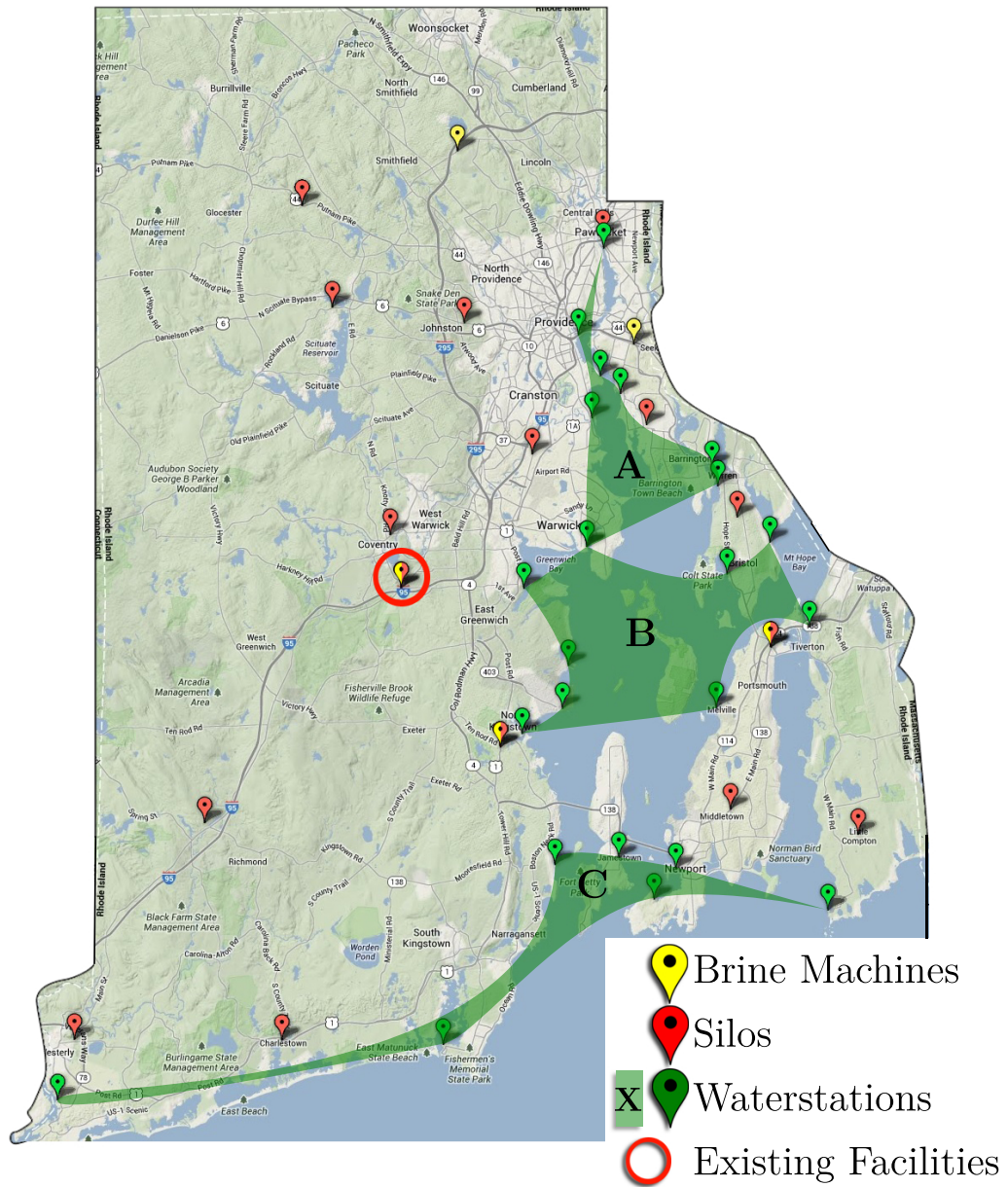


Figure 5.1: Initial situation of RIDOT's facility location problem.

reached from every point in less than 30 miles.[U.S. Geological Survey 2013]

The green tags represent harbours which seem appropriate for an exchange of brine between boat and trucks, where appropriate means that large and heavy trucks can get close enough to the water. An actual analysis of the appropriateness of each access point was not made. Connected access points are grouped to water stations so that an exchange can only take place at these points per period. Not only is this areal restriction important for the model itself, it also caps excessive expenditures for fuel because it is not possible to travel from Providence in the north to Westerly in the south west each period. Moreover, discrete stations provide a better insight into more strategic and less strategic water stations.

RIDOT is interested in a sustainable solution which expedites operations and prevents brine shortage. By investing in silos, brine is evenly distributed over the entire network and consumption miles are reduced. This addresses the former aspect of expediting operations. However, if the accumulated storage capacity is insufficient or the existing brine machine cannot produce enough brine, another brine machine is required to ensure that enough brine is available. Parameters of the model are listed in table 5.1. Realization costs include procurement and installation costs, where installation costs are particularly high for brine machines. These costs include, for example, wiring, plumbing and a concrete pad for the machine to sit on. These parameters are not available for the water alternative so that a sensitivity analysis will identify boundaries of parameter configurations up to which the water alternative is advantageous to conventional approaches.

Table 5.1: Parameters provided by RIDOT.

Facility Type	Realization Costs [\$]	Storage Capacity [gals.]	Production Capacity [gals./period]
Brine Machine	90,000	0	20,000
Silo (small)	10,000	5,000	0
Silo (medium)	15,000	7,500	0
Silo (large)	18,000	10,000	0

5.2 Network Preparation

5.2.1 Street Network

The street network for the FLP is based on the *Rhode Island Geographic Information System* (RIGIS). [Rhode Island Geographic Information System 2013] This network is valuable since it provides information like jurisdiction, number of lanes, and road type for each road segment. This information is required to identify roads maintained by RIDOT, to calculate a segment's demand and to distinguish between primary and secondary roads. However, several problems were encountered by calculating shortest paths. For a start, the network rarely distinguishes between intersections and bridges and shortest paths frequently take not allowed turns. Attempts to determine road segments on the same level failed. The second problem, that is reverse single-directed streets, was even more difficult to solve and results in another problem associated with this network: a set of disjoint road segments in the network. The shortest path should not be calculated based on this network and the network of *OpenStreetMap* is used for this purpose instead. [Open Street Map 2013]

Before distances between prospective facility locations and maintained road segments are determined to reduce the amount of required variables in the model, the network needs to be modified. Distances of road segments are converted into the required time to travel along them, based on

an average velocity contingent to the road type. Thus, not the shortest but the fastest path is calculated, to preferably use highways, and consumption miles are in fact consumption seconds. Since both consumption miles and consumption seconds are artificial units which are only meaningful in relation, the notation consumption miles is, for the sake of clarity, still used in the following.

5.2.2 Creating Flows

For safety reasons, brine trucks cannot spread brine on two opposite directing roads at a time. Therefore, double directed roads are split into two individual, single-directed flows, as described in section 3.1. Based on the road type of the associated roads, flows are then classified as primary or secondary. 34% of lane miles maintained by RIDOT are primary roads, meaning that they have top priority and need to be prepared at all times. These are interstate highways, federal highways, and state highways as well as exits and entries. Secondary flows are associated with town arterials and medium volume roads. Only a specified service level must be maintained. In total, 3,600 primary flows and 18,000 secondary flows exist.

For each of these flows, brine demand and shortest distances to all prospective facility locations are calculated. Since the intensity of a storm does not affect the amount of brine spread on the road, the demand of a flow is constant 50 gallons per lane mile. The scenarios determine when demand occurs, i.e. when a snowstorm hits an area. Brine is applied once per storm. Unlike calculating demand, calculating shortest distances is computationally challenging. For each of the 21,600 flows, the shortest path to all silos and water access points must be determined because demand can be met by those. Additionally, each distance needs to be calculated twice; once with and once without using the flow itself. The idea behind this additional com-

putational burden is that u-turns of trucks shall, when possible, be avoided (see chapter 3). Considering 16 silos and 23 access points, roughly 15.9 million operations are required. In such cases, it is worthwhile to compare different alternatives to obtain the shortest path. As suggested in section 3.1, the Dijkstra algorithm is used. A linear optimization problem is not considered because loading or creating a model takes longer than starting with an algorithm right away. Another advantage of the algorithm is that it incrementally scans the network for the desired point whereas a linear program always considers the entire network. Distances between close-by points are therefore faster to solve with a Dijkstra algorithm.

Two implementations of the algorithm were compared; one using a priority dictionary and the other using a heap queue.[Eppstein 2013 and GitHub 2013] A priority dictionary is a modified dictionary which contains a function to return the shortest path between two points. In contrast, a heap queue saves information in such a way that the point with the shortest path is always the first element. For a comparison, both algorithms were used to calculate the distances between the same 1000 point pairs which are randomly distributed in the network. The result is given in table 5.2. It is shown that the heap queue implementation is, on average, more than twice as fast as the other implementation. The standard deviation is smaller, too, so that the heap queue implementation is chosen for the following operation.

Although the algorithm is fast, it takes more than 50 hours of computing time to calculate all of the distances using a 2.3 GHz single-core

Table 5.2: Comparison of different implementations of the Dijkstra algorithm.

Implementation Type	μ_{1000} [sec]	σ
Priority Dictionary	0.768	0.047
Heap Queue	0.313	0.028

computer. Multiprocessing is applied to reduce operation time even further. In the end, calculating all of the distances took 6.5 hours on a computer with 24 2.3 GHz processors and 32 GB RAM.

5.3 Weather Preparation

The model assumes that all facilities related to brine operations are exclusively used for anti-icing operations and therefore only depend on the weather. In this section, the incorporation of weather records is described.

5.3.1 Processing of Weather Data

Weather records are provided by the *Daily Global Historical Climatology Network*, which is maintained by the *National Oceanic and Atmospheric Administration* (NOAA). [National Oceanic and Atmospheric Administration 2013]

Out of a list of worldwide available weather stations, only those stationed in and around Rhode Island are considered and weather records, dating back to 1980, were downloaded. These records are then converted into a format in which each winter is considered separately, ranging from October 1st to April 30th. The problem encountered with this data is that the quality of information is worse than expected. On average, weather stations had only 11.2% of exploitable information related to snowfall. To improve the quality of information, the time frame considered can be shortened or fewer weather stations can be taken into account. Shortening the considered time frame is avoided because it does not significantly improve the situation (exploitable information since winter 05/06: 18.4%) but reduces the amount of considered weather scenarios tremendously. Instead, fewer weather stations are considered by introducing a 5% threshold of exploitable information. Only 39 of the initial 107 weather stations remain (see figure 5.2). A higher threshold is

not possible since large areas in Rhode Island would be without a representative weather station. In the same figure, grid points are depicted, as well. As described in section 4.1, these grid points represent the weather for close by flows and contribute to a model reduction. After all, assigning 21,600 flows to 39 weather stations for 33 scenarios and 213 periods is difficult and does not necessarily result in a higher quality of information. Especially, when the limiting factor is available weather records. The maximum allowed distance between grid points and weather stations with valid information is 20 kilometers. When no weather station with valid information is available, it is assumed that snowfall did not occur. For data integrity reasons, only weather records provided by NOAA are used.

In figure 5.3, the average annual frequency of snow storms is depicted. The amount of snowfall per snow storm is not considered since it is irrelevant for the problem. Regional differences are obvious and justifies the consideration of weather in the FLP. Although the magnitude of difference between north and south can be called into question, it coincides with the expectation that more snow storms occur inland than at the coast. A reason for this extreme difference is that far fewer weather stations are located in the south and the probability that the assignment procedure cannot assign a grid point to a weather station within the tolerated distance rises. However, since the model is based on the inter-storm days and not on the frequency of snow storms, this problem can be neglected. The reason for this is that the distribution of inter-storm days is of interest and it is assumed that this distribution is not heavily distorted when only some values are missing. The approach of using inter-storm days is chosen because of the premise that the FLP shall address brine shortage in extreme situations, which occurs when storms strike in a row.

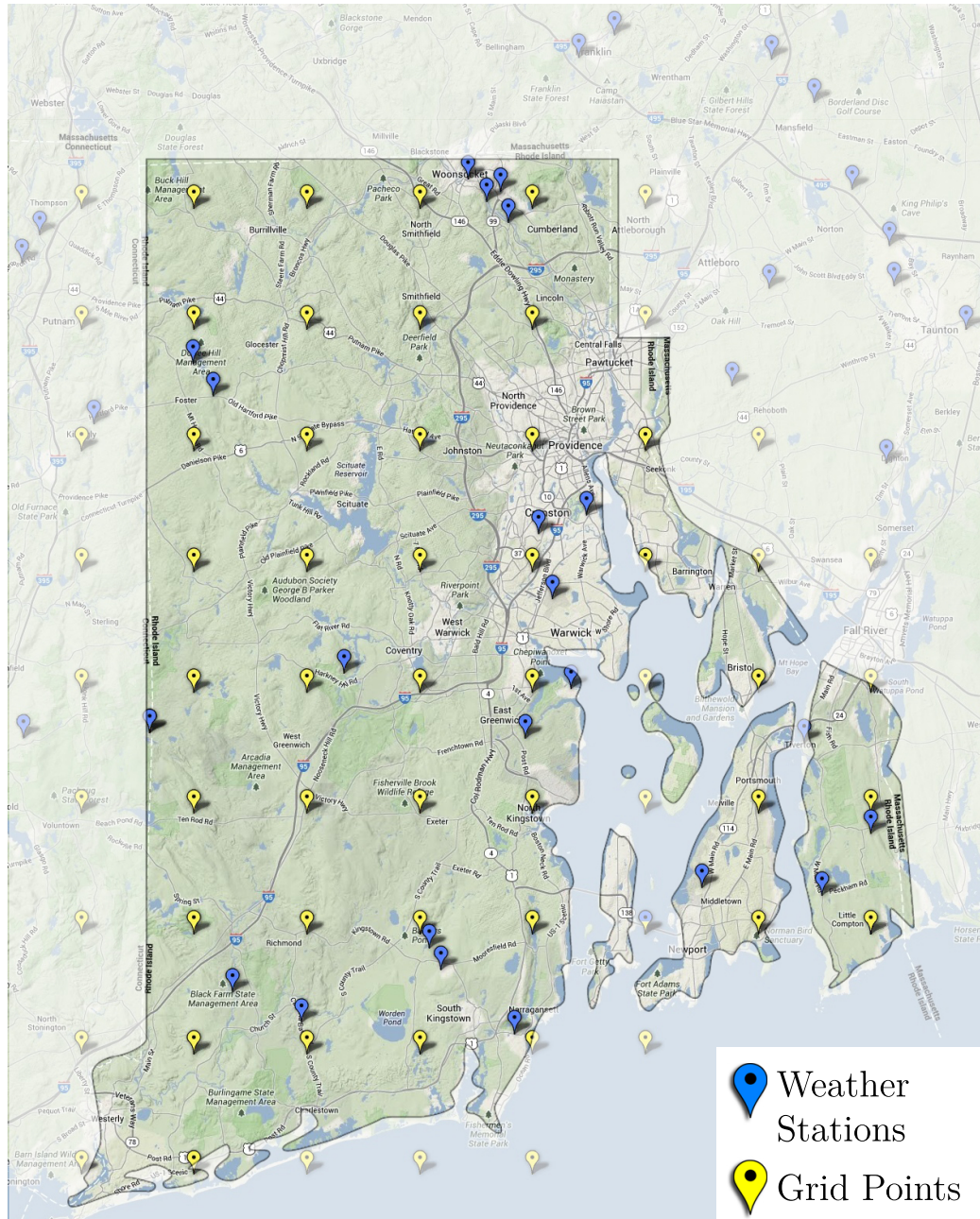


Figure 5.2: Grid points and weather stations in Rhode Island.

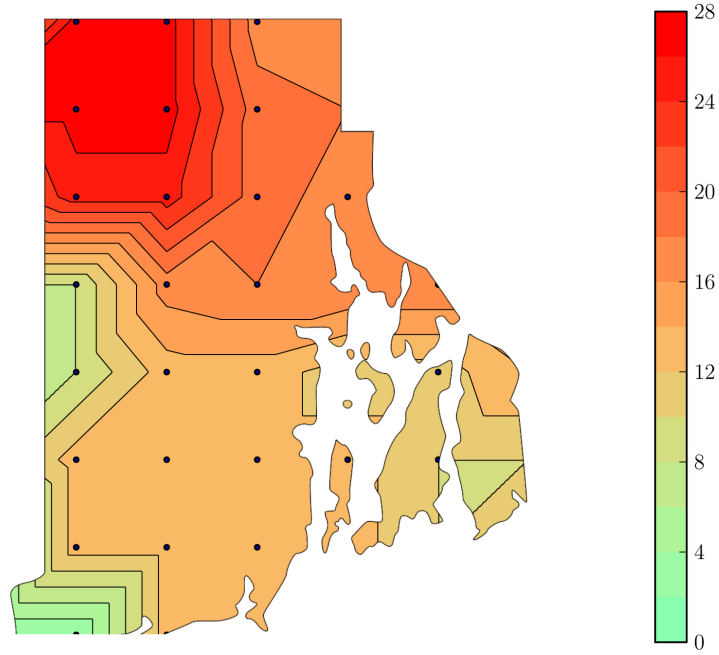


Figure 5.3: Average annual frequency of snow storms in Rhode Island from 1980 to 2013.

5.3.2 *Generating Scenarios*

In order to generate scenarios, a clustering of grid points is required to reduce the amount of considered information. This clustering is based on the quantity of days that two grid points share exactly the same weather events. As depicted in figure 5.4, Rhode Island is separated in three clusters, namely north, middle and south. The prevailing weather conditions in a group are represented by the weather records of a center grid point, which has the least deviation to all other grid points in this group. Logically, the more clusters, the more accurately the center point of a cluster represents the group (figure 5.5). Using many small clusters increases the computational effort because more scenarios must be considered and the model increases heavily in size. Taking the latter aspect into account, only 3 clusters are considered.

Different intensity levels of scenarios, expressed as inter-storm days, are determined as described in table 5.3. First, all days between two snow

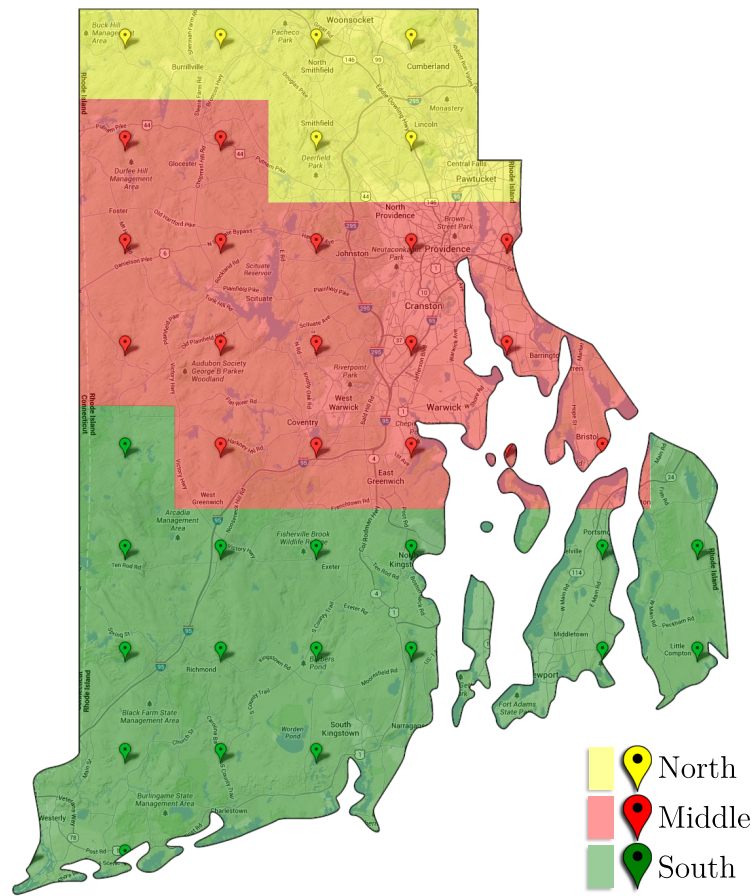


Figure 5.4: Grid point clustering with three centers, based on similar weather events.

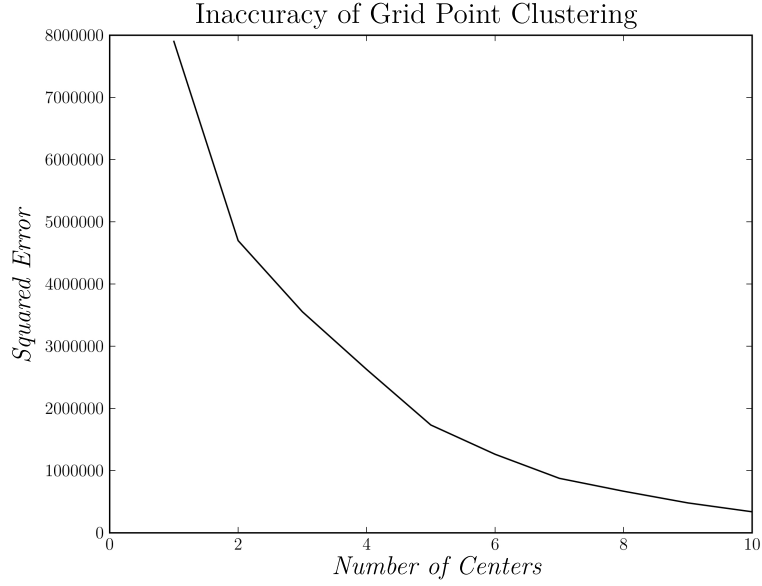


Figure 5.5: Sum of squared days, a grid point does not share the same weather event as its associated center grid point.

storms are determined for each of the 33 considered winters. Only days greater than one and smaller than seven are taken into account because in the former case a subsequent snow storm is assumed and salt is spread instead of brine. In the latter case, it is assumed that the burden of subsequently striking snow storms is interrupted because all silos can be replenished within a week of fair weather. The quantity of inter-storm days is denoted by $y_{w,i}$, where w is the associated winter and i is a consecutive number. For each winter, a discrete distribution of inter-storm days is created and values are determined which are smaller than 95%, 70%, and 40% of all values. These values refer to the quantity of inter-storm days for the high, medium, and low intensity levels for each winter, denoted by $y_{w,h}$, $y_{w,m}$, and $y_{w,l}$, respectively. To obtain three representative quantities for an area, the average of all winters is calculated for each intensity level. Since the model only uses discrete periods, this average, $y_{\bullet,(h/m/l)}$, is eventually adjusted downward. Otherwise, extreme scenarios could not be considered.

Table 5.3: Calculating inter-storm days for scenarios.

Inter-storm days	Intensity Levels		
	High (h)	Medium (m)	Low (l)
$y_{1,1} \dots y_{1,n_1}$	$P(y_{1,h} \leq Y_1)=0,95$	$P(y_{1,m} \leq Y_1)=0,7$	$P(y_{1,l} \leq Y_1)=0,4$
\vdots	\vdots	\vdots	\vdots
$y_{33,1} \dots y_{33,n_{33}}$	$P(y_{33,h} \leq Y_{33})=0,95$	$P(y_{33,m} \leq Y_{33})=0,7$	$P(y_{33,l} \leq Y_{33})=0,4$
$y_{\bullet,h} = \left\lfloor \frac{1}{33} \sum_{s=1}^{33} y_{s,h} \right\rfloor \quad y_{\bullet,m} = \left\lfloor \frac{1}{33} \sum_{s=1}^{33} y_{s,m} \right\rfloor \quad y_{\bullet,l} = \left\lfloor \frac{1}{33} \sum_{s=1}^{33} y_{s,l} \right\rfloor$			

Having calculated three intensity levels for each cluster, scenarios are created by combining all possible intensity level - area combinations. $3^3 = 27$ scenario combinations exist, but only 8 are unique. As described in section 4.1, redundant scenario combinations are merged to avoid unnecessary computational effort. Since all scenarios are considered equally likely, the unique scenarios are weighted with their frequency of occurrence so that the information content remains the same. The following table 5.4 contains all scenario combinations, their frequency and their relative weight. Based on these parameters, scenarios are created, meaning that for a period of 10 days, snow storms are simulated every time the number of respective inter-storm days elapse.

Table 5.4: Unique scenarios and their frequency of occurrence.

Scenario	Inter-storm days			Frequency	rel. Weight
	North	Middle	South		
1	3	3	3	2	0.074
2	2	2	4	4	0.148
3	2	3	3	4	0.148
4	3	3	4	1	0.038
5	3	2	3	4	0.148
6	2	3	4	2	0.074
7	3	2	4	2	0.074
8	2	2	3	8	0.296

5.4 Results

The facility type mix shall be chosen so that brine shortage is avoided as much as possible. Using only the existing brine machine and the two existing silos located with it, demand of primary roads cannot be met in either of the considered scenarios. Thus, the primary goal of the investment is to ensure that at least all primary flows can be treated with brine; secondary flows do not necessarily need to be treated. As it turns out, an investment budget of \$100,000 is large enough so that 100% of all secondary flows can be prepared, as well, and no slack is allowed in the following analysis.

The FLP is solved based on a minimum regret model. Therefore, two steps are required: a scenario-dependent facility mix with which an optimal solution can be obtained and a scenario-independent facility mix which minimizes the deviation from the optimum in all scenarios. Because of the large size of the model, an optimality gap of 5% is used throughout the analysis.

5.4.1 *Scenario-dependent Solutions*

The water alternative is a novelty and parameters do not yet exist. A sensitivity analysis is applied in the following to determine boundaries of parameters for which this alternative is feasible. Referring to RIDOT, the question remains whether a realization within these boundaries is feasible. Only two parameters are subject to change: realization costs and production capacity. Storage capacity is assumed to be 5,000 gallons, which corresponds to the storage capacity of a small silo.

Case I The water alternative is assumed to be as good and as expensive as a regular brine machine. The result shows that investments in a small silo

and in the water alternative are the best solution in regard to minimizing consumption miles. It is less surprising that the water alternative is more advantageous than the brine machine because of the mobility aspect.

In 7 out of 8 scenarios, the silo is placed in the south west region of Rhode Island, either in Westerly or in Charlestown; in the remaining scenario, the silo is placed in Pawtucket, north of Providence. In 70.3% of the time a water station delivers brine to road segments or silos, water station A (north) is utilized. In the remaining 29.7%, brine was delivered from water station C (south west). The result is depicted in figure 5.6, whereas the width of each bin equals its weight.

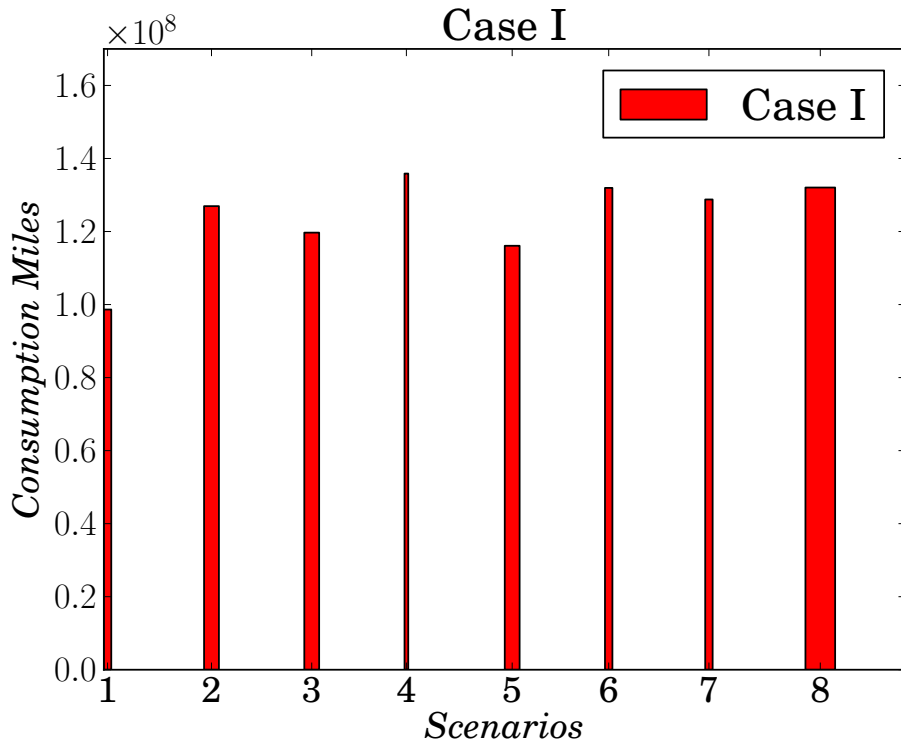


Figure 5.6: Water alternative is as expensive and produces as much brine as a brine machine. The solution comprises the water alternative and an additional silo at different locations for each scenario.

Case II In the second case, it is assumed that the water alternative is as expensive as a brine machine and a silo; its production capacity remains the same. Solving the model a second time shows that the water alternative is again the best investment, meaning that the mobility advantage is at least worth \$10,000. In comparison to Case I, the overall consumption miles increased by 8.8%, due to the missing additional silo. This time, the water alternative delivers brine to road segments and silos in 84% of the time from water station A and only in 16% of the time from water station C. The result is depicted in figure 5.7.

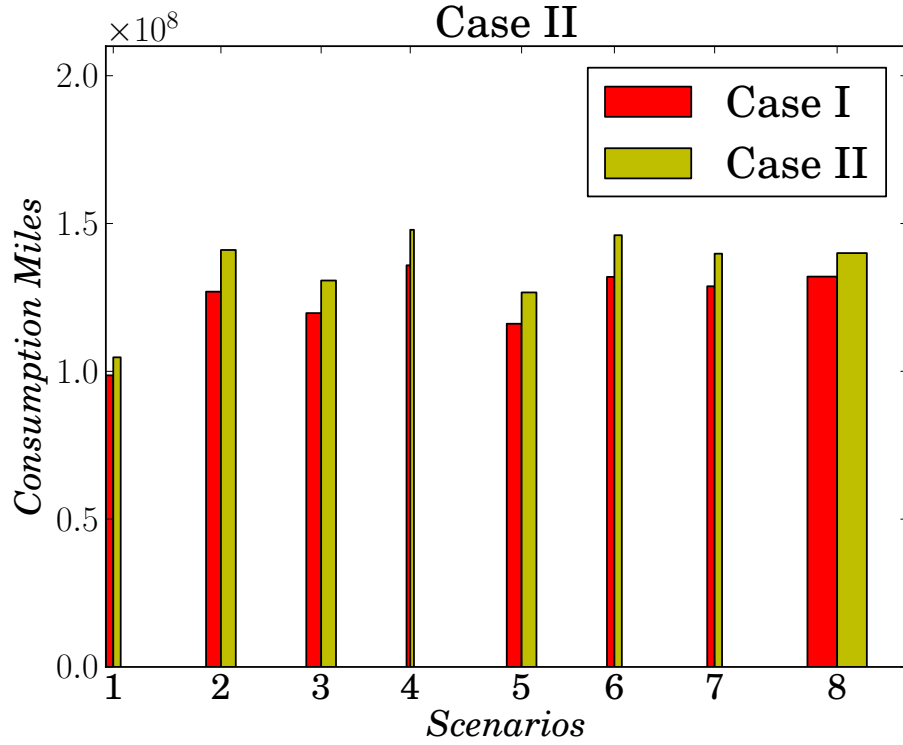


Figure 5.7: Water alternative is as expensive as a brine machine and a silo. Only the water alternative is part of the solution.

Case III By incrementally decreasing the production capacity of the water alternative, the threshold of 18,000 gallons appears to be significant. Below

this level, the model is not feasible because the water alternative cannot produce enough brine for the entire street network. Thus, the water alternative is only an option when a production capacity of at least 18,000 gallons can be ensured. Below this threshold, the model suggests to invest in silos, not in another brine machine, and it can be inferred that the accumulated storage capacity of additional silos is enough to withstand any intensity level for a planning horizon of 10 periods. Figure 5.8 shows the result of Case III. The weighted sum of consumption miles of Case III is 26.6% worse than Case II.

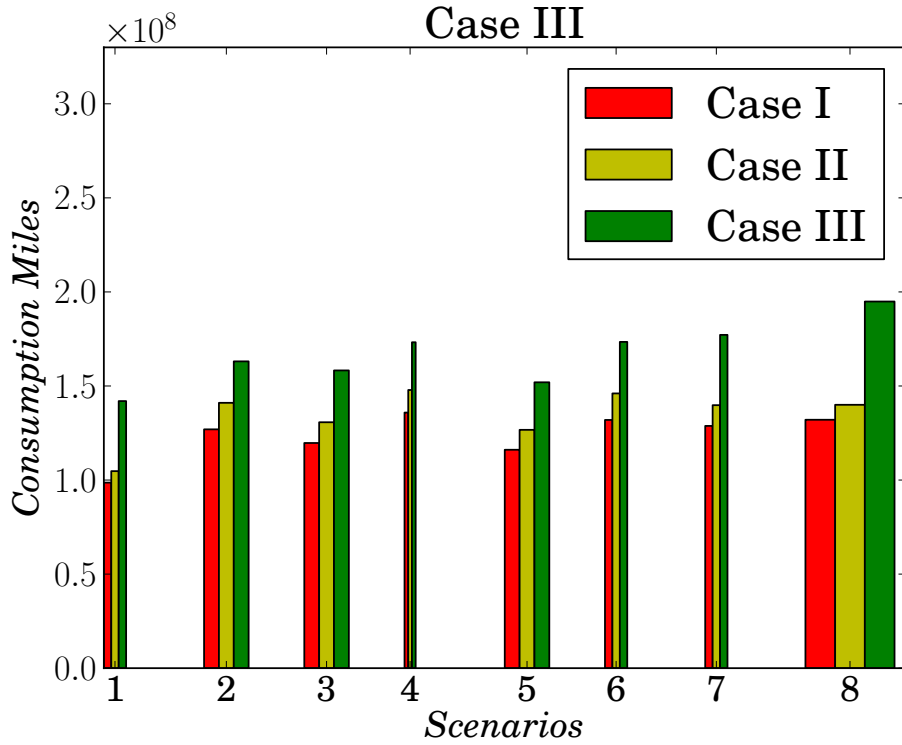


Figure 5.8: Production capacity of water alternative is below allowed minimum. Investments in silos are made instead.

For the analysis of the scenario-independent FLP, it is of interest to see how the location of silos changes in comparison to the optimal locations for each scenario. The investment in a brine machine can be neglected from this point forth since the accumulated storage capacity of silos is sufficient to

meet all demands and investing in several silos is more advantageous in reducing consumption miles than investing in a brine machine. In figure 5.9, all activated silos are depicted. The size of each tag represents the percentage of activated capacity at a prospective silo location over all scenarios compared to the maximum possible one. The maximum possible one is 80,000 gals., i.e. when a large silo with a capacity of 10,000 gals. is assigned to a prospective location in all 8 scenarios. For example, in scenarios 2 and 4 a silo with 10,000 gallons capacity is activated at the prospective silo location in Westerly (south west). In scenarios 3,5,6,7, and 8, only a small silo is activated and in scenario 1, no silo is activated. In numbers, only 55.6% of the maximum possible capacity of 80,000 gals. are part of the optimal facility location decisions across all scenarios.

The height of the bar below each tag represents the amount of brine delivered to this silo. Referring to the location in Westerly, it can be inferred that this silo location is strategically good enough to be utilized to roughly 50% capacity but it is not replenished at all. It is located too far in the south west to actually be worth replenishing. Those locations are considered satellite stations and they are inferior to equally utilized ones which, in contrast, are replenished. These silos are part of the solution due to the assumption that all silos, when activated, are initially filled to capacity. But since each silo needs to be replenished eventually, satellite silos are not necessarily advantageous.

5.4.2 *Scenario-independent Solutions*

Determining a scenario-independent solution is computationally challenging. Therefore, parameters of the *IBM ILOG CPLEX Optimization Studio* were adjusted to increase the solver's performance. For instance, binary variables were prioritized. Activating silos or the water alternative are on one level,

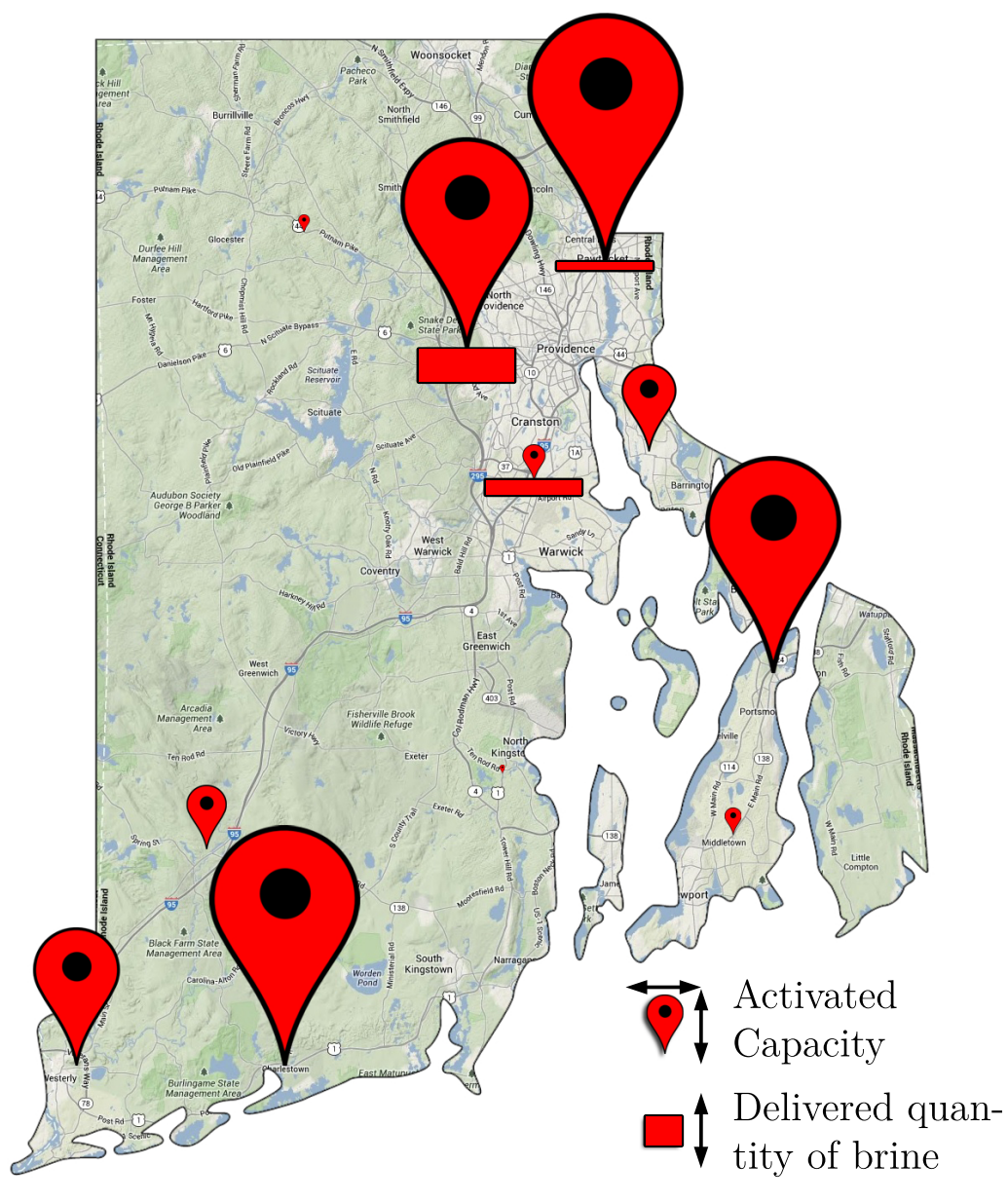


Figure 5.9: Overview of activated storage facility locations and associated storage capacities across all scenarios (Case III).

activating additional storage capacity or water station are on a lower level. In the following, the solver has an upper time limit of 2.5 days; an optimality gap is not specified.

Case IIa In subsection 5.4.1, a minimum brine production threshold for the water alternative is given. Solving the scenario-independent model with a production above this threshold results in the same results as presented in Case II. After all, the best water stations for each scenario can still be activated and the regret is zero.

Case IIIa Since it is already proven that the water alternative is the best choice, it is neglected in this case to reduce the size of the model. However, the model is still considerably large and the solver terminates with an optimality gap of 10.9%. As depicted in figure 5.10, the overall weighted consumption miles are 34.1% worse compared to Case II/IIa. The expected regret to Case III is 5.9%. The silo locations for the scenario-independent FLP are depicted in figure 5.11. Three 10,000 gallon silos are assigned to the locations in Pawtucket (1), Johnston (2), and Charlestown (4) and two 7,500 gallon silos are assigned to the locations in Portsmouth (3) and Westerly (5). This facility mix seems to be reasonable because these silos are activated the most across all scenarios in Case III (see figure 5.9). The silo in Johnston has the most strategic location. It is replenished several times, in contrast to the satellite silos in Westerly and Charlestown.

5.5 Discussion and Recommendations

From the results obtained above, it already becomes apparent that the water alternative is the best choice for reducing consumption miles and therefore expediting operations. Using a storage capacity of 5,000 gallons, the pro-

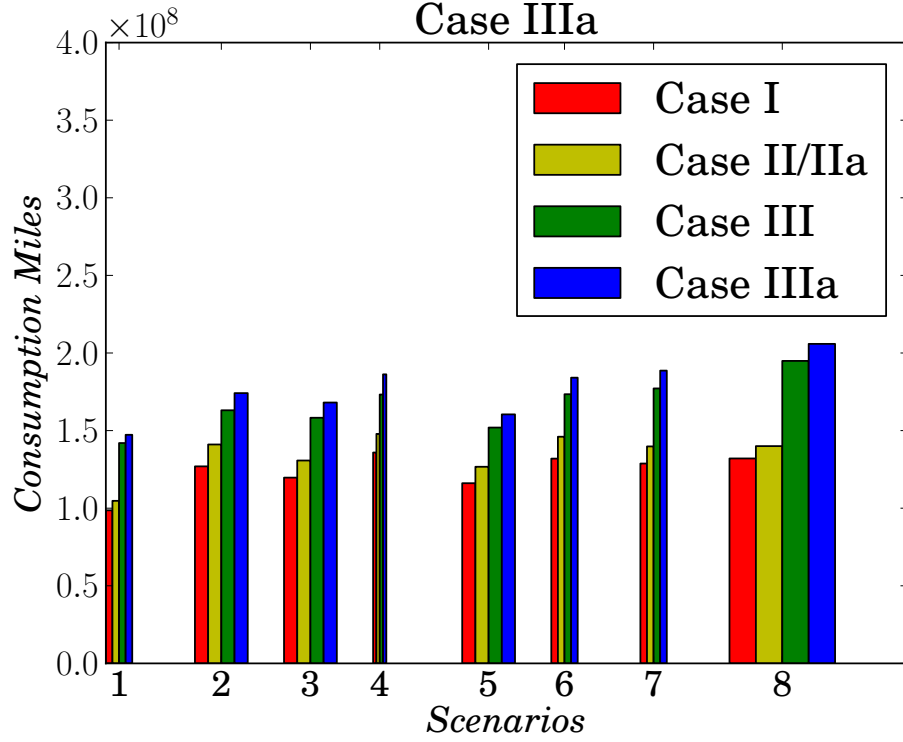


Figure 5.10: Scenario-dependent results with a scenario-independent facility location decision. The facility mix consists of silos with different storage capacities.

duction capacity of the water alternative must be at least 18,000 gallons per day. Below this threshold, not enough brine can be provided to meet all demand. A lower production capacity can certainly be offset by a higher storage capacity, but this relation is not covered in this work. This information becomes useful when the implementation of the water alternative moves forward.

Despite an optimality gap of 11%, the result for the scenario-independent FLP seems reasonable. Its validity is emphasized by the fact that those locations are quite often part of the scenario-dependent optimal solution and it is unlikely that the locations of silos will change when using more than 2.5 days computation time.

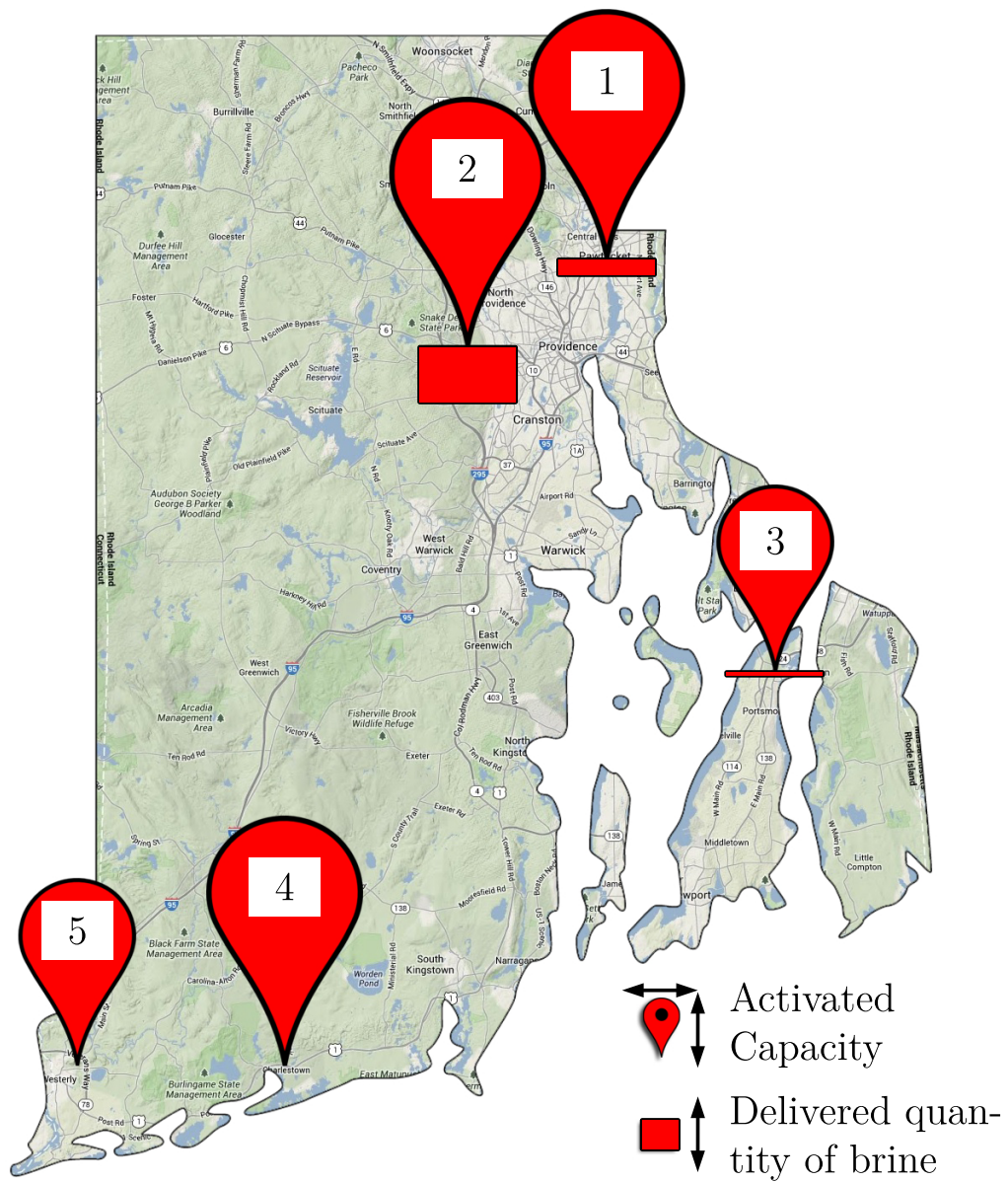


Figure 5.11: Overview of the scenario-independent facility location decision obtained with an optimality gap of 10.9% (Case IIIa).

CONCLUSIONS AND FUTURE WORK

6.1 Summary and Conclusions

This work addresses a facility location problem (FLP) for augmenting existing snow removal operations with brine spreading operations. Brine is a salt water mixture which is sprayed on roads prior to snowfall. It turns snow into slush and prevents it from packing to the asphalt. Using brine is advantageous to salt because operational expenses and environmental pollution can be reduced. Brine ensures that salt sticks to the asphalt so that it can be applied more economically. In addition, less salt is actually spread since it is distributed more evenly over the asphalt when dissolved in water.

To augment existing operations, investments in either storage or production facilities are required and their locations in a network need to be determined. In contrast to conventional FLPs for snow removal, the FLP in this work does not aim to determine the most central locations in a network. Instead, the benefit of assigning facilities to locations is assessed based on the first premise stated in chapter 1, namely that the facilities shall expedite spreading operations. Since facilities serve as replenishment points for spreading trucks, dead mileage is reduced and time is saved. The second premise refers to the facility type mix and addresses the likelihood of brine shortage, which is equivalent to the failure of the system. Investments in silos and brine machines are considered. Silos are significantly less expensive than

brine machines so that investments in multiple silos is advantageous in regard to the first premise since more replenishment points can be set up. However, the storage capacity of silos is finite and they need to be replenished every now and then. When no brine is available in the system, a silo is useless and it fails to meet its purpose. In other words, the intention of setting up a silo was good but the solution is impractical. Investments in brine machines should be made instead.

In addition to finding a trade-off between setting up many replenishment points and averting brine shortage, a novel concept is introduced, which addresses both premises: a brine machine is placed on a boat. This boat travels along the coast and serves as a need-based replenishment point.

In chapter 3, the deterministic mathematical model for determining beneficial locations is presented. In chapter 4, this model is extended by a variable aspect and weather records are incorporated to address the selection of an optimal facility type mix. Since it is assumed that brine machines, silos, and the water alternative are only utilized for brine operations, weather is the only considered source of variability.

The objective of the deterministic model is to site facilities such that consumption miles are minimized. Consumption miles is an artificial unit which corresponds to the amount of brine required by a road segment times the distance travelled to deliver that brine from the source to the sink. Consequently, the demand of every road segment is met individually by either a silo or the water alternative. The reason for this approach is that truck routes cannot be anticipated since trucks do not start and end their operations from the facilities to be placed. As it becomes apparent in the case study for Rhode Island's Department of Transportation in chapter 5, meeting demand individually increases the size of the model and preprocessing techniques are applied. Therefore, shortest distances between sources and

sinks are calculated beforehand and are fed into the model as parameters.

The likelihood of brine shortage is addressed by creating a stress test of differently intense scenarios for a prospective facility type mix. Only those facility mixes are considered which ensure a feasible solution in all scenarios. A scenario involves different areas of the network experiencing storms in certain intervals, or so-called inter-storm days. For a given planning horizon, demand in these areas is generated every time the number of inter-storm days elapses. The system is modelled as a regret model.

In chapter 5, the devised model is applied to a case study for the Rhode Island Department of Transportation. The results show that the mobility aspect of the water alternative is beneficial, particularly in regard to the scenario-independent model because the optimal water stations for each scenario can always be utilized. The regret is zero. Solving the minimum regret model without the water alternative shows that the accumulated capacity of all silos is large enough to withstand the brine demand in all scenarios. Investments in brine machines are consequently not suggested because setting up several silos decreases consumption miles more effectively than setting up only one brine machine. The scenario-independent solution obtained after 2.5 days of computing shows that setting up large silos with 10,000 gallon capacity in Pawtucket, Johnston, and Charlestown and setting up medium silos with 7,500 gallon capacity in Portsmouth and Westerly is most beneficial.

6.2 Critical Appraisal and Future Work

Obviously, the result of the FLP can be distorted when the quality of weather information is poor. It is consequently recommended to solve the model with alternative weather records to validate results. Without a comparison, this is almost impossible, due to the large amount of information. Since only

fee-based weather records were found, except for those provided by NOAA, this validation was not possible in this work.

Based on the model presented in this work, the water alternative is advantageous to conventional silos and brine machines. However, the analysis of this work does not make a statement regarding whether this water alternative is actually feasible and further studies are required to assess its practicability. This would also include to identify water access points and the maximum storage and production capacity. It also remains to assess whether operating a brine machine on a boat is indeed efficient. It might turn out that this alternative is far too expensive and savings due to the usage of salt water does not compensate for higher operating costs. In contrast, when the water alternative is efficient, further research is required to maximize its potential. For instance, synchronizing truck and boat routes so that instant replenishment is possible is a difficult task. Such a routing problem must be solved on an operational level, based on the weather forecast, and truck and boat routes would need to meet at water access points to exchange brine every now and then. Using several trucks operating in different areas of a network would make it even more difficult because travel time of a boat between two water access points plus the time required to replenish a truck needs to be considered. Poorly synchronized operations in which a truck spends more time waiting than spreading should be avoided.

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